Contents

Preface xi

1 Introduction 1
1.1 What are knapsack problems? 1
1.2 Terminology 2
1.3 Computational complexity 6
1.4 Lower and upper bounds 9

2 0-1 Knapsack problem 13
2.1 Introduction 13
2.2 Relaxations and upper bounds 16
2.2.1 Linear programming relaxation and Dantzig's bound 16
2.2.2 Finding the critical item in $O(n)$ time 17
2.2.3 Lagrangian relaxation 19
2.3 Improved bounds 20
2.3.1 Bounds from additional constraints 20
2.3.2 Bounds from Lagrangian relaxations 23
2.3.3 Bounds from partial enumeration 24
2.4 The greedy algorithm 27
2.5 Branch-and-bound algorithms 29
2.5.1 The Horowitz–Sahni algorithm 30
2.5.2 The Martello–Toth algorithm 32
2.6 Dynamic programming algorithms 36
2.6.1 Elimination of dominated states 39
2.6.2 The Horowitz–Sahni algorithm 43
2.6.3 The Toth algorithm 44
2.7 Reduction algorithms 45
2.8 Approximate algorithms 50
2.8.1 Polynomial-time approximation schemes 50
2.8.2 Fully polynomial-time approximation schemes 53
2.8.3 Probabilistic analysis 56
2.9 Exact algorithms for large-size problems 57
2.9.1 The Balas–Zemel algorithm 58
2.9.2 The Fayard–Plateau algorithm 60
2.9.3 The Martello–Toth algorithm 61
2.10 Computational experiments 67
2.10.1 Exact algorithms 68
2.10.2 Approximate algorithms 71
2.11 Facets of the knapsack polytope 74
2.12 The multiple-choice knapsack problem 77

3 Bounded knapsack problem **81**
3.1 Introduction 81
3.2 Transformation into a 0-1 knapsack problem 82
3.3 Upper bounds and approximate algorithms 84
3.3.1 Upper bounds 84
3.3.2 Approximate algorithms 86
3.4 Exact algorithms 87
3.4.1 Dynamic programming 88
3.4.2 Branch-and-bound 88
3.5 Computational experiments 89
3.6 A special case: the unbounded knapsack problem 91
3.6.1 Upper bounds and approximate algorithms 92
3.6.2 Exact algorithms 95
3.6.3 An exact algorithm for large-size problems 98
3.6.4 Computational experiments 102

4 Subset-sum problem **105**
4.1 Introduction 105
4.2 Exact algorithms 106
4.2.1 Dynamic programming 106
4.2.2 A hybrid algorithm 109
4.2.3 An algorithm for large-size problems 116
4.3 Approximate algorithms 117
4.3.1 Greedy algorithms 117
4.3.2 Polynomial-time approximation schemes 120
4.3.3 Fully polynomial-time approximation schemes 125
4.3.4 Probabilistic analysis 126
4.4 Computational experiments 128
4.4.1 Exact algorithms 129
4.4.2 Approximate algorithms 130

5 Change-making problem **137**
5.1 Introduction 137
5.2 Lower bounds 138
5.3 Greedy algorithms 140
5.4 When the greedy algorithm solves classes of knapsack problems 142
5.5 Exact algorithms 145
5.5.1 Dynamic programming 145
5.5.2 Branch-and-bound 146
5.6 An exact algorithm for large-size problems 149
5.7 Computational experiments 151
5.8 The bounded change-making problem 153

6 0-1 Multiple knapsack problem **157**
6.1 Introduction 157
6.2 Relaxations and upper bounds 158
6.2.1 Surrogate relaxation 158
6.2.2 Lagrangian relaxation 162
6.2.3 Worst-case performance of the upper bounds 165
6.3 Greedy algorithms 166
6.4 Exact algorithms 167
6.4.1 Branch-and-bound algorithms 168
6.4.2 The "bound-and-bound" method 170

6.4.3 A bound-and-bound algorithm 172
6.5 Reduction algorithms 176
6.6 Approximate algorithms 177
6.6.1 On the existence of approximation schemes 177
6.6.2 Polynomial-time approximation algorithms 179
6.7 Computational experiments 182

7 Generalized assignment problem 189
7.1 Introduction 189
7.2 Relaxations and upper bounds 192
7.2.1 Relaxation of the capacity constraints 192
7.2.2 Relaxation of the semi-assignment constraints 195
7.2.3 The multiplier adjustment method 197
7.2.4 The variable splitting method 201
7.3 Exact algorithms 204
7.4 Approximate algorithms 206
7.5 Reduction algorithms 209
7.6 Computational experiments 213

8 Bin-packing problem 221
8.1 Introduction 221
8.2 A brief outline of approximate algorithms 222
8.3 Lower bounds 224
8.3.1 Relaxations based lower bounds 224
8.3.2 A stronger lower bound 228
8.4 Reduction algorithms 233
8.5 Exact algorithms 237
8.6 Computational experiments 240

Appendix: Computer codes 247
A.1 Introduction 247
A.2 0-1 Knapsack problem 248
A.2.1 Code MT1 248
A.2.2 Code MT1R 249
A.2.3 Code MT2 251
A.3 Bounded and unbounded knapsack problem 252
A.3.1 Code MTB2 252
A.3.2 Code MTU2 254
A.4 Subset-sum problem 256
A.4.1 Code MTSL 256
A.5 Bounded and unbounded change-making problem 258
A.5.1 Code MTC2 258
A.5.2 Code MTCB 259
A.6 0-1 Multiple knapsack problem 261
A.6.1 Code MTM 261
A.6.2 Code MTHM 263
A.7 Generalized assignment problem 265
A.7.1 Code MTG 265
A.7.2 Code MTHG 268
A.8 Bin-packing problem 270
A.8.1 Code MTP 270

Glossary 273

Bibliography 275

Author index 283

Subject index 287