

Contents

FOREWORD	xxi
PREFACE	xxxiii
DEFINITION OF SYMBOLS	xxxvii
1 THE LINEAR PROGRAMMING PROBLEM	1
1.1 SOME SIMPLE EXAMPLES	2
1.2 MATHEMATICAL STATEMENT	7
1.3 FORMULATING LINEAR PROGRAMS	8
1.3.1 The Column (Recipe/Activity) Approach	9
1.3.2 The Row (Material Balance) Approach	11
1.4 EXAMPLES OF MODEL FORMULATION	12
1.4.1 Product Mix Problem (Column Approach)	12
1.4.2 Product Mix Problem (Row Approach)	15
1.4.3 A Simple Warehouse Problem	16
1.4.4 On-the-Job Training	18
1.5 BOUNDS	21
1.6 AXIOMS	22
1.7 NOTES & SELECTED BIBLIOGRAPHY	23
1.8 PROBLEMS	25
2 SOLVING SIMPLE LINEAR PROGRAMS	35
2.1 TWO-VARIABLE PROBLEM	35
2.2 TWO-EQUATION PROBLEM	37
2.2.1 Graphical Solution	38
2.2.2 The Dual Linear Program	41
2.3 FOURIER-MOTZKIN ELIMINATION	43
2.3.1 Illustration of the FME Process	44
2.3.2 The Fourier-Motzkin Elimination Algorithm	46
2.3.3 Fourier-Motzkin Elimination Theory	47
2.4 INFEASIBILITY THEOREM	52
2.5 NOTES & SELECTED BIBLIOGRAPHY	53

2.6	PROBLEMS	54
3	THE SIMPLEX METHOD	63
3.1	GRAPHICAL ILLUSTRATION	64
3.2	THE SIMPLEX ALGORITHM	64
3.2.1	Canonical Form and Basic Variables	64
3.2.2	Improving a Nonoptimal Basic Feasible Solution	68
3.2.3	The Simplex Algorithm	71
3.2.4	Theory Behind the Simplex Algorithm	73
3.3	SIMPLEX METHOD	76
3.3.1	The Method	77
3.3.2	Phase I/Phase II Algorithm	78
3.3.3	Theory Behind Phase I	81
3.4	BOUNDED VARIABLES	83
3.5	REVISED SIMPLEX METHOD	89
3.5.1	Motivation	89
3.5.2	Revised Simplex Method Illustrated	92
3.5.3	Revised Simplex Algorithm	93
3.5.4	Computational Remarks	96
3.6	NOTES & SELECTED BIBLIOGRAPHY	97
3.7	PROBLEMS	98
4	INTERIOR-POINT METHODS	113
4.1	BASIC CONCEPTS	115
4.2	PRIMAL AFFINE / DIKIN'S METHOD	118
4.3	INITIAL SOLUTION	121
4.4	NOTES & SELECTED BIBLIOGRAPHY	122
4.5	PROBLEMS	124
5	DUALITY	129
5.1	DUAL AND PRIMAL PROBLEMS	129
5.1.1	Von Neumann Symmetric Form	129
5.1.2	Tucker Diagram	130
5.1.3	Duals of Mixed Systems	130
5.1.4	The Dual of the Standard Form	132
5.1.5	Primal-Dual Feasible-Infeasible Cases	133
5.2	DUALITY THEOREMS	134
5.3	COMPLEMENTARY SLACKNESS	135
5.4	OBTAINING A DUAL SOLUTION	136
5.5	NOTES & SELECTED BIBLIOGRAPHY	138
5.6	PROBLEMS	139

6	EQUIVALENT FORMULATIONS	145
6.1	RESTRICTED VARIABLES	145
6.2	UNRESTRICTED (FREE) VARIABLES	146
6.3	ABSOLUTE VALUES	147
6.4	GOAL PROGRAMMING	150
6.5	MINIMIZING THE MAXIMUM OF LINEAR FUNCTIONS	152
6.6	CURVE FITTING	154
6.7	PIECEWISE LINEAR APPROXIMATIONS	157
6.7.1	Convex/Concave Functions	157
6.7.2	Piecewise Continuous Linear Functions	159
6.7.3	Separable Piecewise Continuous Linear Functions	160
6.8	NOTES & SELECTED BIBLIOGRAPHY	162
6.9	PROBLEMS	162
7	PRICE MECHANISM AND SENSITIVITY ANALYSIS	171
7.1	THE PRICE MECHANISM OF THE SIMPLEX METHOD	172
7.1.1	Marginal Values or Shadow Prices	173
7.1.2	Economic Interpretation of the Simplex Method	174
7.1.3	The Manager of a Machine Tool Plant	175
7.1.4	The Ambitious Industrialist	181
7.1.5	Sign Convention on Prices	183
7.2	INTRODUCING A NEW VARIABLE	184
7.3	INTRODUCING A NEW CONSTRAINT	186
7.4	COST RANGING	188
7.5	CHANGES IN THE RIGHT-HAND SIDE	190
7.6	CHANGES IN THE COEFFICIENT MATRIX	192
7.7	THE SUBSTITUTION EFFECT OF NONBASIC ACTIVITIES ON BASIC ACTIVITIES	198
7.8	NOTES AND SELECTED BIBLIOGRAPHY	199
7.9	PROBLEMS	199
8	TRANSPORTATION AND ASSIGNMENT PROBLEM	205
8.1	THE CLASSICAL TRANSPORTATION PROBLEM	205
8.1.1	Mathematical Statement	206
8.1.2	Properties of the System	206
8.2	STANDARD TRANSPORTATION ARRAY	212
8.3	FINDING AN INITIAL SOLUTION	214
8.3.1	Triangularity Rule	214
8.3.2	The Least Remaining Cost Rule	217
8.3.3	Vogel's Approximation Method	217
8.3.4	Russel's Approximation Method	218
8.3.5	Cost Preprocessing	219
8.4	FAST SIMPLEX ALGORITHM FOR THE TRANSPORTATION PROBLEM	222
8.4.1	Simplex Multipliers, Optimality, and the Dual	222

8.4.2	Finding a Better Basic Solution	224
8.4.3	Illustration of the Solution Process	225
8.5	THE ASSIGNMENT PROBLEM	229
8.6	EXCESS AND SHORTAGE	233
8.6.1	Mathematical Statement	234
8.6.2	Properties of the System	236
8.6.3	Conversion to the Classical Form	236
8.6.4	Simplex Multipliers and Reduced Costs	238
8.7	PRE-FIXED VALUES AND INADMISSIBLE SQUARES	239
8.8	THE CAPACITATED TRANSPORTATION PROBLEM	240
8.9	NOTES & SELECTED BIBLIOGRAPHY	244
8.10	PROBLEMS	245
9	NETWORK FLOW THEORY	253
9.1	TERMINOLOGY	253
9.2	FLOWS AND ARC-CAPACITIES	258
9.3	AUGMENTING PATH ALGORITHM FOR MAXIMAL FLOW	262
9.4	CUTS IN A NETWORK	275
9.5	SHORTEST ROUTE	277
9.6	MINIMAL SPANNING TREE	282
9.7	MINIMUM COST-FLOW PROBLEM	286
9.8	THE NETWORK SIMPLEX METHOD	288
9.9	THE BOUNDED VARIABLE PROBLEM	299
9.10	NOTES & SELECTED BIBLIOGRAPHY	301
9.11	PROBLEMS	304
A	LINEAR ALGEBRA	315
A.1	SCALARS, VECTORS, AND MATRICES	315
A.2	ARITHMETIC OPERATIONS WITH VECTORS AND MATRICES	317
A.3	LINEAR INDEPENDENCE	320
A.4	ORTHOGONALITY	321
A.5	NORMS	321
A.6	VECTOR SPACES	324
A.7	RANK OF A MATRIX	326
A.8	MATRICES WITH SPECIAL STRUCTURE	326
A.9	INVERSE OF A MATRIX	329
A.10	INVERSES OF SPECIAL MATRICES	330
A.11	DETERMINANTS	331
A.12	EIGENVALUES	333
A.13	POSITIVE-DEFINITENESS	336
A.14	NOTES & SELECTED BIBLIOGRAPHY	337
A.15	PROBLEMS	337

B	LINEAR EQUATIONS	341
B.1	SOLUTION SETS	341
B.2	SYSTEMS OF EQUATIONS WITH THE SAME SOLUTION SETS	343
B.3	HOW SYSTEMS ARE SOLVED	345
B.4	ELEMENTARY OPERATIONS	346
B.5	CANONICAL FORMS, PIVOTING, AND SOLUTIONS	349
B.6	PIVOT THEORY	354
B.7	NOTES & SELECTED BIBLIOGRAPHY	357
B.8	PROBLEMS	357
	REFERENCES	361