

Contents

Part I Models

1	Introduction and Examples	3
1.1	A Farming Example and the News Vendor Problem	4
a.	The farmer's problem	4
b.	A scenario representation	6
c.	General model formulation	10
d.	Continuous random variables	11
e.	The news vendor problem	15
1.2	Financial Planning and Control	20
1.3	Capacity Expansion	28
1.4	Design for Manufacturing Quality	35
1.5	A Routing Example	40
a.	Presentation	40
b.	Wait-and-see solutions	42
c.	Expected value solution	43
d.	Recourse solution	44
e.	Other random variables	46
f.	Chance-constraints	47
1.6	Other Applications	48
2	Uncertainty and Modeling Issues	55
2.1	Probability Spaces and Random Variables	55
2.2	Deterministic Linear Programs	57
2.3	Decisions and Stages	57
2.4	Two-Stage Program with Fixed Recourse	59
a.	Fixed distribution pattern, fixed demand, r_i, v_j, t_{ij} stochastic	62
b.	Fixed distribution pattern, uncertain demand	63
c.	Uncertain demand, variable distribution pattern	64
d.	Stages versus periods; Two-stage versus multistage	65

2.5	Random Variables and Risk Aversion	66
2.6	Implicit Representation of the Second Stage	68
	a. A closed form expression is available for $\mathcal{Q}(x)$	69
	b. For a given x , $\mathcal{Q}(x)$ is computable	70
2.7	Probabilistic Programming	71
	a. Deterministic linear equivalent: a direct case	71
	b. Deterministic linear equivalent: an indirect case	72
	c. Deterministic nonlinear equivalent: the case of random constraint coefficients	73
2.8	Modeling Exercise	74
	a. Presentation	74
	b. Discussion of solutions	76
2.9	Alternative Characterizations and Robust Formulations	84
2.10	Relationship to Other Decision-Making Models	87
	a. Statistical decision theory and decision analysis	87
	b. Dynamic programming and Markov decision processes	89
	c. Machine learning and online optimization	90
	d. Optimal stochastic control	91
	e. Summary	93
2.11	Short Reviews	94
	a. Linear programming	94
	b. Duality for linear programs	96
	c. Nonlinear programming and convex analysis	97

Part II Basic Properties

3	Basic Properties and Theory	103
3.1	Two-Stage Stochastic Linear Programs with Fixed Recourse	103
	a. Formulation	103
	b. Discrete random variables	105
	c. General cases	109
	d. Special cases: relatively complete, complete, and simple recourse	113
	e. Optimality conditions and duality	115
	f. Stability and nonanticipativity	118
3.2	Probabilistic or Chance Constraints	124
	a. General case	124
	b. Probabilistic constraints with discrete random variables	130
3.3	Stochastic Integer Programs	135
	a. Recourse problems	135
	b. Simple integer recourse	140
	c. Probabilistic constraints	146
3.4	Multistage Stochastic Programs with Recourse	149
3.5	Stochastic Nonlinear Programs with Recourse	156

4 The Value of Information and the Stochastic Solution 163

4.1 The Expected Value of Perfect Information 163

4.2 The Value of the Stochastic Solution 165

4.3 Basic Inequalities 166

4.4 The Relationship between *EVPI* and *VSS* 167

 a. $EVPI = 0$ and $VSS \neq 0$ 168

 b. $VSS = 0$ and $EVPI \neq 0$ 169

4.5 Examples 170

4.6 Bounds on *EVPI* and *VSS* 171

Part III Solution Methods

5 Two-Stage Recourse Problems 181

5.1 The *L*-Shaped Method 182

 a. Optimality cuts 184

 b. Feasibility cuts 191

 c. Proof of convergence 196

 d. The multicut version 198

5.2 Regularized Decomposition 202

5.3 The Piecewise Quadratic Form of the *L*-shaped Methods 210

5.4 Bunching and Other Efficiencies 217

 a. Full decomposability 218

 b. Bunching 219

5.5 Basis Factorization and Interior Point Methods 222

5.6 Inner Linearization Methods and Special Structures 237

5.7 Simple and Network Recourse Problems 242

5.8 Methods Based on the Stochastic Program Lagrangian 253

5.9 Additional Methods and Complexity Results 262

6 Multistage Stochastic Programs 265

6.1 Nested Decomposition Procedures 266

6.2 Quadratic Nested Decomposition 276

6.3 Block Separability and Special Structure 282

6.4 Lagrangian-Based Methods for Multiple Stages 284

7 Stochastic Integer Programs 289

7.1 Stochastic Integer Programs and LP-Relaxation 289

7.2 First-stage Binary Variables 291

 a. Improved optimality cuts 294

 b. Example with continuous random variables 299

7.3 Second-stage Integer Variables 302

 a. Looking in the space of tenders 303

 b. Discontinuity points 305

 c. Algorithm 306

7.4 Reformulation 312

 a. Difficulties of reformulation in stochastic integer programs . 312

b.	Disjunctive cuts	314
c.	First-stage dependence	316
d.	An algorithm	317
7.5	Simple Integer Recourse	319
a.	χ restricted to be integer	322
b.	The case where $S = 1$, χ not integral	325
7.6	Cuts Based on Branching in the Second Stage	326
a.	Feasibility cuts	326
b.	Optimality cuts	329
7.7	Extensive Forms and Decomposition	331
7.8	Short Reviews	334
a.	Branch-and-bound	334
b.	A simple example of valid inequalities	335
c.	Disjunctive cuts	336

Part IV Approximation and Sampling Methods

8	Evaluating and Approximating Expectations	341
8.1	Direct Solutions with Multiple Integration	342
8.2	Discrete Bounding Approximations	346
8.3	Using Bounds in Algorithms	352
8.4	Bounds in Chance-Constrained Problems	357
8.5	Generalized Bounds	363
a.	Extensions of basic bounds	363
b.	Bounds based on separable functions	367
c.	General-moment bounds	372
8.6	General Convergence Properties	381
9	Monte Carlo Methods	389
9.1	Sample Average Approximation and Importance Sampling in the L -Shaped Method	390
9.2	Stochastic Decomposition	395
9.3	Stochastic Quasi-Gradient Methods	399
9.4	Sampling Methods for Probabilistic Constraints and Quantiles	404
9.5	General Results for Sample Average Approximation and Sequential Sampling	409
10	Multistage Approximations	417
10.1	Extensions of the Jensen and Edmundson-Madansky Inequalities ..	418
10.2	Bounds Based on Aggregation	422
10.3	Scenario Generation and Distribution Fitting	426
10.4	Multistage Sampling and Decomposition Methods	432
10.5	Approximate Dynamic Programming and Special Cases	436
a.	Network revenue management	438
b.	Vehicle allocation problems	439
c.	Piecewise-linear separable bounds	441

- d. Nonlinear bounds and a production planning example 444
- e. Extensions 446
- Sample Distribution Functions** 449
 - A.1 Discrete Random Variables 449
 - A.2 Continuous Random Variables 450
- References** 451
- Author Index** 471
- Subject Index** 477