## **Contents**

Pref	ace			iii
1	Stochastic Programming Models			
	1.1	Introduction	n	1
	1.2			1
		1.2.1	The Newsvendor Problem	1
		1.2.2	Chance Constraints	5
		1.2.3	Multistage Models	6
	1.3	Multi-Produ	uct Assembly	9
		1.3.1	Two Stage Model	9
		1.3.2	Chance Constrained Model	11
		1.3.3	Multistage Model	12
	1.4	Portfolio Se	election	13
		1.4.1	Static Model	13
		1.4.2	Multistage Portfolio Selection	17
		1.4.3	Decision Rules	21
	1.5	Supply Cha	nin Network Design	22
	Exercis	ses		25
2	Two Stage Problems			27
	2.1	_	Stage Problems	27
		2.1.1	Basic Properties	27
		2.1.2	The Expected Recourse Cost for Discrete Distributions .	30
		2.1.3	The Expected Recourse Cost for General Distributions	33
		2.1.4	Optimality Conditions	38
	2.2	Polyhedral	Two-Stage Problems	42
		2.2.1	General Properties	42
		2.2.2	Expected Recourse Cost	44
		2.2.3	Optimality Conditions	47
	2.3	General Two-Stage Problems		48
		2.3.1	Problem Formulation, Interchangeability	48
		2.3.2	Convex Two-Stage Problems	50
	2.4	Nonanticipa	ativity	53
		2.4.1	Scenario Formulation	53

<u>viii</u> Contents

		2.4.2	Dualization of Nonanticipativity Constraints 5		
		2.4.3	Nonanticipativity Duality for General Distributions 5		
		2.4.4	Value of Perfect Information 6		
	Exerc	cises			
3		Multistage Problems 6			
	3.1		Formulation		
		3.1.1	The General Setting 6		
		3.1.2	The Linear Case		
		3.1.3	Scenario Trees		
		3.1.4	Filtration Interpretation		
		3.1.5	Algebraic Formulation of Nonanticipativity Constraints . 7.		
		3.1.6	Piecewise Affine Policies		
	3.2	Duality			
		3.2.1	Convex Multistage Problems 8		
		3.2.2	Optimality Conditions		
		3.2.3	Dualization of Feasibility Constraints 8		
		3.2.4	Dualization of Nonanticipativity Constraints 8		
	Exerc	cises			
4	Optin	mization M	Iodels with Probabilistic Constraints 9		
	4.1		tion		
	4.2	Convexit	ty in probabilistic optimization		
		4.2.1	Generalized concavity of functions and measures 10		
		4.2.2	Convexity of probabilistically constrained sets 11		
		4.2.3	Connectedness of probabilistically constrained sets 12		
	4.3	Separabl	le probabilistic constraints		
		4.3.1	Continuity and differentiability properties of distribution		
			functions		
		4.3.2	p-Efficient points		
		4.3.3	The tangent and normal cones of conv $\mathbb{Z}_p$		
		4.3.4	Optimality conditions and duality theory		
	4.4	Optimiza	ation problems with non-separable probabilistic constraints 15		
		4.4.1	Differentiability of probability functions and optimality		
			conditions		
		4.4.2	Approximations of non-separable probabilistic constraints 15		
	4.5	Semi-inf	finite probabilistic problems		
	Exerc				
5	Statistical Inference 17:				
	5.1		al Properties of SAA Estimators		
	3.53.539	5.1.1	Consistency of SAA Estimators		
		5.1.2	Asymptotics of the SAA Optimal Value		
		5.1.3	Second Order Asymptotics		
		5.1.4	Minimax Stochastic Programs		
	5.2		ic Generalized Equations		
		SOUTH			

Contents İх

		5.2.1	Consistency of Solutions of the SAA Generalized Equa-			
			tions			
		5.2.2	Asymptotics of SAA Generalized Equations Estimators			
	5.3		Monte Carlo Sampling Methods			
		5.3.1	Exponential Rates of Convergence and Sample Size Es-	10000		
			timates in Case of Finite Feasible Set			
		5.3.2	Sample Size Estimates in General Case			
	<b>.</b> .	5.3.3	Finite Exponential Convergence			
	5.4		nte Carlo Methods			
	5.5		Reduction Techniques			
		5.5.1	Latin Hypercube Sampling			
		5.5.2	Linear Control Random Variables Method			
		5.5.3	Importance Sampling and Likelihood Ratio Methods			
	5.6		Analysis			
		5.6.1	Estimation of the Optimality Gap			
		5.6.2	Statistical Testing of Optimality Conditions			
	5.7		onstrained Problems			
		5.7.1	Monte Carlo Sampling Approach			
		5.7.2	Validation of an Optimal Solution			
	5.8		nod Applied to Multistage Stochastic Programming			
		5.8.1	Statistical Properties of Multistage SAA Estimators			
		5.8.2	Complexity Estimates of Multistage Programs			
	5.9		Approximation Method			
		5.9.1	Classical Approach			
		5.9.2	Robust SA Approach			
		5.9.3	Mirror Descent SA Method			
		5.9.4	Accuracy Certificates for Mirror Descent SA Solutions .			
	5.10		Dual Dynamic Programming Method			
		5.10.1	Approximate Dynamic Programming Approach			
		5.10.2	The SDDP algorithm			
		5.10.3	Convergence Properties of the SDDP Algorithm			
		5.10.4	Risk Averse SDDP Method			
	Exercis	ses		. 285		
6	Risk A	verse Optir		289		
	6.1		on			
	6.2		models			
			Main ideas of mean-risk analysis			
		6.2.2	Semideviations			
		6.2.3	Weighted mean deviations from quantiles			
		6.2.4	Average Value-at-Risk			
	6.3	Coherent I	Risk Measures			
		6.3.1	Differentiability Properties of Risk Measures			
		6.3.2	Examples of Risk Measures			
		6.3.3	Law Invariant Risk Measures	. 318		
		6.3.4	Spectral Risk Measures	. 323		

Contents

		6.3.5	Kusuoka Representations	. 327
		6.3.6	Probability Spaces with Atoms	. 334
		6.3.7	Stochastic Orders	. 337
	6.4	Ambiguo	us Chance Constraints	. 339
	6.5	Optimiza	tion of Risk Measures	. 347
		6.5.1	Dualization of Nonanticipativity Constraints	. 350
		6.5.2	Interchangeability Principle for Risk Measures	. 351
		6.5.3	Examples	. 354
	6.6	Statistical	l Properties of Risk Measures	. 359
		6.6.1	Average Value-at-Risk	. 359
		6.6.2	Absolute Semideviation Risk Measure	. 364
		6.6.3	Von Mises Statistical Functionals	. 366
	6.7	The Prob	lem of Moments	. 369
	6.8	Multistag	e Risk Averse Optimization	. 374
		6.8.1	Scenario Tree Formulation	. 374
		6.8.2	Conditional Risk Mappings	. 381
		6.8.3	Dynamic Risk Measures	
		6.8.4	Risk Averse Multistage Stochastic Programming	
		6.8.5	Time Consistency of Multiperiod Problems	
		6.8.6	Minimax Approach to Risk-Averse Multistage Program-	
			ming	
		6.8.7	Portfolio Selection and Inventory Model Examples	
	Exerc	ises		
7		ground Mat		413
	7.1	600 To 100	tion and Convex Analysis	
		7.1.1	Directional Differentiability	
		7.1.2	Elements of Convex Analysis	
		7.1.3	Optimization and Duality	
		7.1.4	Optimality Conditions	
		7.1.5	Perturbation Analysis	. 432
		7.1.6	Epiconvergence	. 440
	7.2	Probabilit	ty	. 441
		7.2.1	Probability Spaces and Random Variables	. 441
		7.2.2	Conditional Probability and Conditional Expectation	. 446
		7.2.3	Measurable Multifunctions and Random Functions	. 448
		7.2.4	Expectation Functions	. 451
		7.2.5	Uniform Laws of Large Numbers	. 457
		7.2.6	Law of Large Numbers for Risk Measures	. 462
		7.2.7	Law of Large Numbers for Random Sets and Subdiffer-	
			entials	
		7.2.8	Delta Method	
		7.2.9	Exponential Bounds of the Large Deviations Theory	
		7.2.10	Uniform Exponential Bounds	
	7.3		of Functional Analysis	
		7.3.1	Conjugate Duality and Differentiability	