OptimizationCity

ENCYCLOPEDIA OF OPERATIONS RESEARCH

Course 3: Duality Theory and Dual Simplex Method

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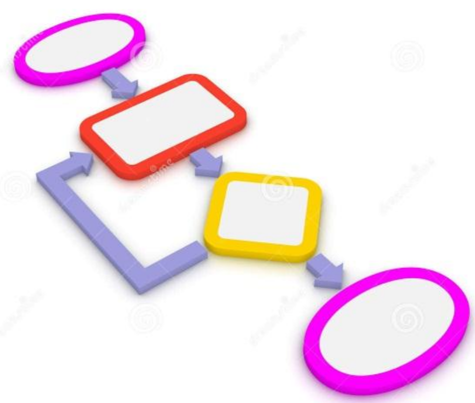
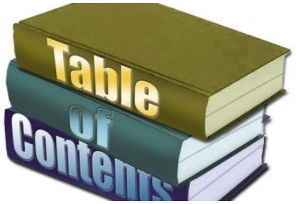


فهرست مطالب



Introduction

Introduction

One of the most important discoveries in the early development of linear programming was the concept of duality and its many important specification. This discovery revealed that every linear programming problem has associated with it another linear programming problem called **the dual**. The relationships between the dual problem and the primal problem prove to be extremely useful in a variety of ways. To clarify the subject, consider the model as follow:

$$Max\ Z = \sum_{j=1}^{n} c_j x_j$$

$s.t.$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad i = 1, \dots, m$$

$$x_j \ge 0 \quad j = 1, \dots, n.$$

Introduction

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$$Min\ y_0 = \sum_{i=1}^{m} b_i y_j$$

$s.t.$

$$\sum_{i=1}^{m} a_{ij} y_i \ge c_j \quad j = 1, \ldots, n.$$

$$y_i \ge 0 \quad i = 1, \ldots, m.$$

Writing the dual problem

The necessary steps to write the dual problem are given below. Whenever the primary problem does not have an equality constraint or a unrestricted variable, it will be as follows:

Step 1) If the objective function of the primal problem is maximization, we make the constraints of the dual problem less or equal.

Step 2) If the constraints of the primary problem are less or equal, then the constraints of the dual problem will be less or equal, too.

Step 3) For each constraint in the primary problem, we consider a variable in the dual problem.

Step 4) The coefficients of the objective function of the dual problem are formed from the numbers on the right hand side of the primary problem.

Writing the dual problem

Step 5) The numbers on the right hand side of the constraints of the dual problem are obtained from the coefficients of the objective function of the primary problem.

Step 6) All the variables of the primary and dual problem are non-negative.

Writing the dual problem

Example: Write the dual model of the following model.

$$
\begin{aligned}
Max \quad Z &= 4x_1 + 5x_2 \\
x_1 + 2x_2 &\le 10 \\
6x_1 + 6x_2 &\le 36 \\
8x_1 + 4x_2 &\le 40 \\
x_1, x_2 &\ge 0
\end{aligned}
$$

Writing the dual problem

$$Max\ Z = \boxed{4}\ x_1 + \boxed{5}\ x_2$$

$$\begin{aligned} \boxed{1}\ x_1 + \boxed{2}\ x_2 &\leq \boxed{10} \\ \boxed{6}\ x_1 + \boxed{6}\ x_2 &\leq \boxed{36} \\ \boxed{8}\ x_1 + \boxed{4}\ x_2 &\leq \boxed{40} \end{aligned}$$

$$x_1, x_2 \geq 0$$

$$Min\ Y = \boxed{10}\ y_1 + \boxed{36}\ y_2 + \boxed{40}\ y_3$$

$$\begin{aligned} \boxed{1}\ y_1 + \boxed{6}\ y_2 + \boxed{8}\ y_3 &\geq \boxed{4} \\ \boxed{2}\ y_1 + \boxed{6}\ y_2 + \boxed{4}\ y_3 &\geq \boxed{5} \end{aligned}$$

$$y_1, y_2, y_3 \geq 0$$

Dual problem for other forms

Dual problem for other forms

If the initial problem has constraints in the form of equality, each constraint in the form of equality is replaced by two constraints with the same variables and coefficients, one greater or equal and the other smaller or equal. Then by multiplying the greater or equal constraint by minus one, both constraints will be smaller or equal.

Dual problem for other forms

Example: write the dual model of the following problem.

$$Max\ Z = x_1 + 2x_2$$
$$2x_1 + x_2 = 5$$
$$3x_1 - x_2 \le 6$$
$$x_1, x_2 \ge 0$$

Solution: The first constraint is equivalent to the following two inequality constraints:

$$2x_1 + x_2 = 5 \rightarrow \begin{cases} 2x_1 + x_2 \le 5 \\ 2x_1 + x_2 \ge 5 \rightarrow -2x_1 - x_2 \le -5 \end{cases}$$

Dual problem for other forms

Now the problem becomes like this:

$$Max\ Z = x_1 + 2x_2$$

$$2x_1 + x_2 \leq 5$$

$$-2x_1 - x_2 \leq -5$$

$$3x_1 - x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

If we denote the variable of the dual problem with the above three constraints by y_1, y_2 , and y_3; the dual problem will be as follows.

$$Min\ Y = 5y_1 - 5y_2 + 6y_3$$

$$2y_1 - 2y_2 + 3y_3 \geq 1$$

$$y_1 - y_2 - y_3 \geq 2$$

$$y_1, y_2, y_3 \geq 0$$

Dual problem for other forms

By changing the above problem, $y_4=y_1-y_2$, y_4 becomes a unrestricted variable. Because the result of the difference of two non-negative values y_2 and y_1 can be positive or negative, so we have:

$$\begin{aligned} Min\,Y &= 5y_4 + 6y_3 \\ & 2y_4 + 3y_3 \geq 1 \\ & y_4 - y_3 \geq 2 \\ & y_3 \geq 0, y_4 : Unrestricted \end{aligned}$$

So as a rule we can say:

Whenever an equality constraint is established in the primary problem, the variable corresponding to that constraint in the dual problem is unrestriced.

Dual problem for other forms

Example: Write the dual model of the following problem.

$$
\begin{aligned}
Max\ Z &= 3x_1 + x_2 \\
& 2x_1 + x_2 \leq 4 \\
& 3x_1 - x_2 \leq 6 \\
& x_2 \geq 0, x_1 \text{:Unrestricted}
\end{aligned}
$$

Solution:

We change the unrestriced x_1 as x_2-x_3.

$$
\begin{aligned}
Max\ Z &= 3x_3 - 3x_4 + x_2 \\
& 2x_3 - 2x_4 + x_2 \leq 4 \\
& 3x_3 - 3x_4 - x_2 \leq 6 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Dual problem for other forms

$$
\begin{aligned}
Min\,Y &= 4y_1 + 6y_2 \\
& 2y_1 + 3y_2 \geq 3 \\
& -2y_1 - 3y_2 \geq -3 \\
& y_1 - y_2 \geq 1 \\
& y_1, y_2 \geq 0
\end{aligned}
$$

The first constraint can be replaced by the equality constraint. The final form of the problem is as follows:

$$
\begin{aligned}
Min\,Y &= 4y_1 + 6y_2 \\
& 2y_1 + 3y_2 = 3 \\
& y_1 - y_2 \geq 1 \\
& y_1, y_2 \geq 0
\end{aligned}
$$

So as a rule we can say:

Whenever a variable is unrestriced in the primary problem, the contraint of that variable in the dual problem is equality.

Dual problem for other forms

Example: Consider the following model:

$$Max\ Z = 3x_1 + 5x_2$$

$$s.t.$$

$$\begin{array}{llll} (1) & x_1 & \leq 4 & (y_1) \\ (2) & 2x_2 & \leq 12 & (y_2) \\ (3) & 3x_1 + 2x_2 & \leq 18 & (y_3) \\ & x_i \geq 0 \quad i = 1, 2. & & \end{array}$$

Construct the dual problem.

Dual problem for other forms

Solution:

$$Min\ y_0 = 4y_1 + 12y_2 + 18y_3$$

$$s.t.$$

$$(1) \quad y_1 \qquad + 3y_3 \geq 3$$

$$(2) \quad 2y_2 \ + 2y_2 \geq 5$$

$$y_i \geq 0 \quad i = 1, 2, 3.$$

Dual problem for other forms

Model	Decision variables	Shadow prices
Primal model	$x_1 = 2$ $x_2 = 6$	$y_1 = 0$ $y_2 = 1.5$ $y_3 = 1$
Dual model	$y_1 = 0$ $y_2 = 1.5$ $y_3 = 1$	$s_1 = 2$ $s_2 = 6$

Observation

Dual problem for other forms

	Primal model			Dual model	
Row	Basic solution	Feasible?	Objective function	Basic solution	Feasible?
1	(0,0,4,12,18)	Yes	0	(0,0,0,-3,-5)	No
2	(4,0,0,12,6)	Yes	12	(3,0,0,0,-5)	No
3	(6,0,-2,12,0)	No	18	(0,0,1,0,-3)	No
4	(4,3,0,6,0)	Yes	27	(-4.5,0,2.5,0,0)	No
5	(0,6,4,0,6)	Yes	30	(0,2.5,0,-3,0)	No
6	(2,6,2,0,0)	Yes	36	(0,1.5,1,0,0)	Yes
7	(4,6,0,0,-6)	No	42	(3,2.5,0,0,0)	Yes
8	(0,9,4,-6,0)	No	45	(0,0,2.5,4.5,0)	Yes

Relation of primal and dual models

Relationships between the solutions of the primal and dual problems

Dual problem		Primary problem
Bounded optimal solution	↔	Bounded optimal solution
No optimal solution	←	Unbounded optimal solution
No optimal solution		No optimal solution
Unbounded optimal solution		

Relation of primal and dual models

Example: Construct and graph a primal problem with two decision variables and two functional constraints that has feasible solutions and an unbounded objective function. Then construct the dual problem and demonstrate graphically that it has no feasible solutions.

Solution:

The model is as follows:

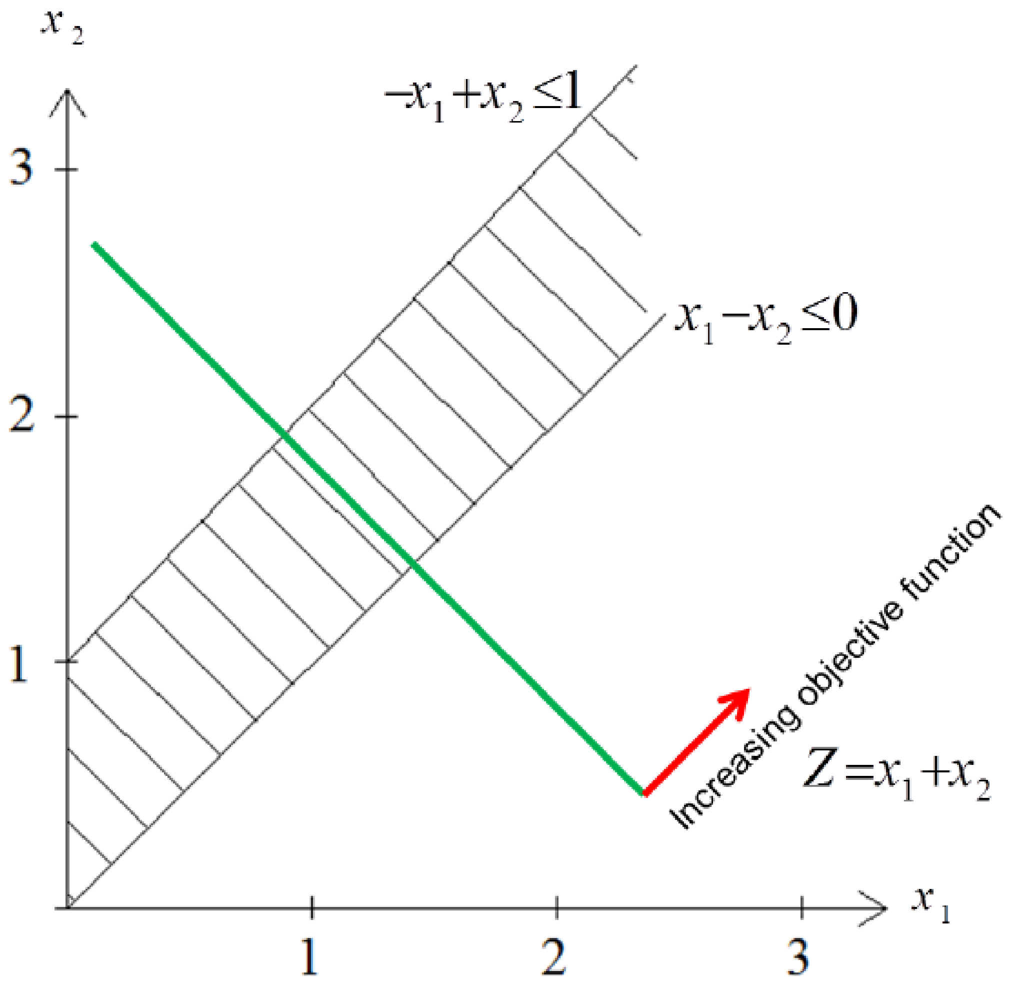
$$
\begin{aligned}
&Max\ Z = x_1 + x_2 \\
&s.t. \\
&\qquad (1) \quad -x_1 + x_2 \le 1 \qquad (y_1) \\
&\qquad (2) \quad\ \ x_1 - x_2 \le 0 \qquad (y_2) \\
&\qquad\qquad x_i \ge 0 \quad i = 1, 2.
\end{aligned}
$$

Relation of primal and dual models

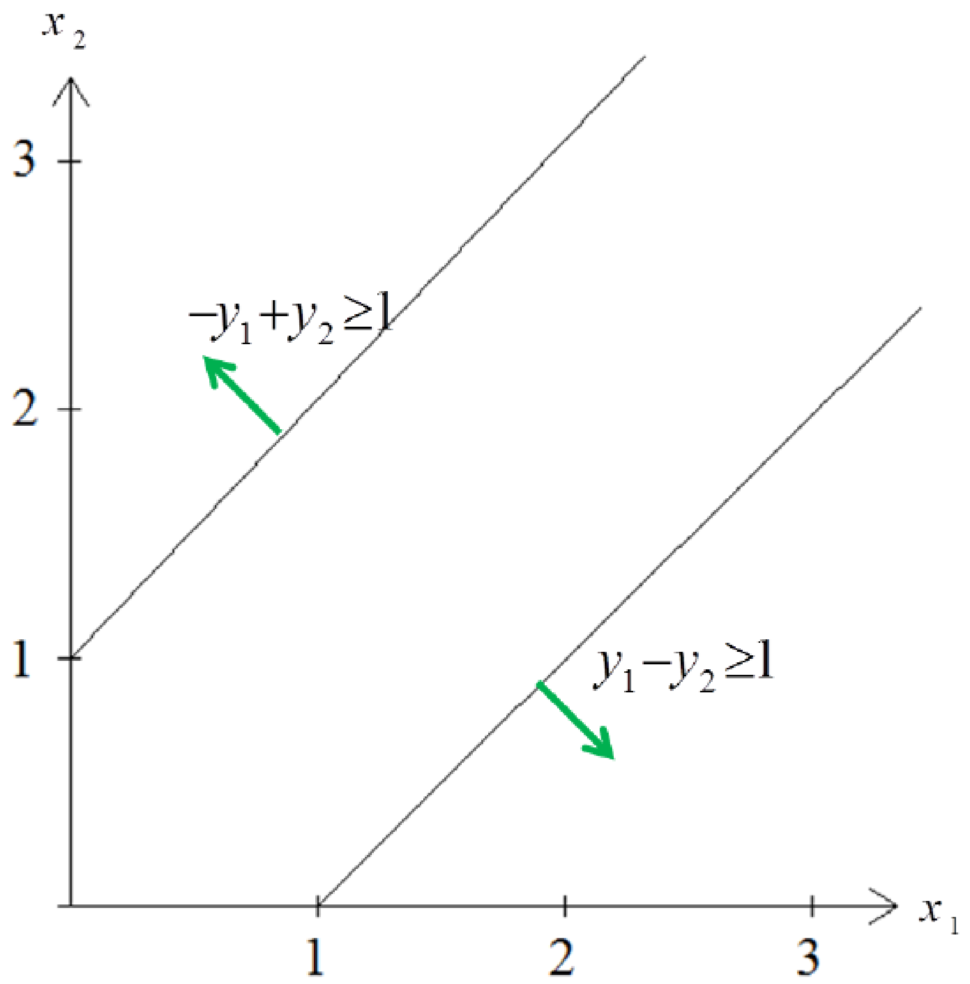
If $x_1 = x_2 = c$ and c approaches to infinity, then $Z = 2c$ and therefore the objective function approaches to infinity. The Dual problem is as follows.

$$
\begin{aligned}
& Min\ y_0 = y_1 \\
& s.t. \\
& \qquad (1) \quad -y_1 + y_2 \geq 1 \\
& \qquad (2) \quad \ \ y_1 - y_2 \geq 1 \\
& \qquad\qquad y_i \geq 0 \quad i = 1, 2.
\end{aligned}
$$

Relation of primal and dual models



Relation of primal and dual models



The dual problem is infeasible.

Relation of primal and dual models

Example: Consider the following problem.

$$Max \quad Z = 6x_1 + 8x_2$$

$s.t.$

$$(1) \quad 5x_1 + 2x_2 \le 20$$

$$(2) \quad x_1 + 2x_2 \le 10$$

$$x_i \ge 0 \quad i = 1, 2.$$

a) Construct the dual problem for this primal problem.

b) Solve both the primal problem and the dual problem graphically.

c) Use the information obtained in part (b) to construct a table listing the complementary basic solutions for these problems.

d) Work through the simplex method step by step to solve the primal problem. After each iteration (including iteration 0), identify the basic feasbile solution for this problem and the complementary basic solution for the dual problem.

Relation of primal and dual models

Solution:

a)

$$Min \quad W = 20y_1 + 10y_2$$

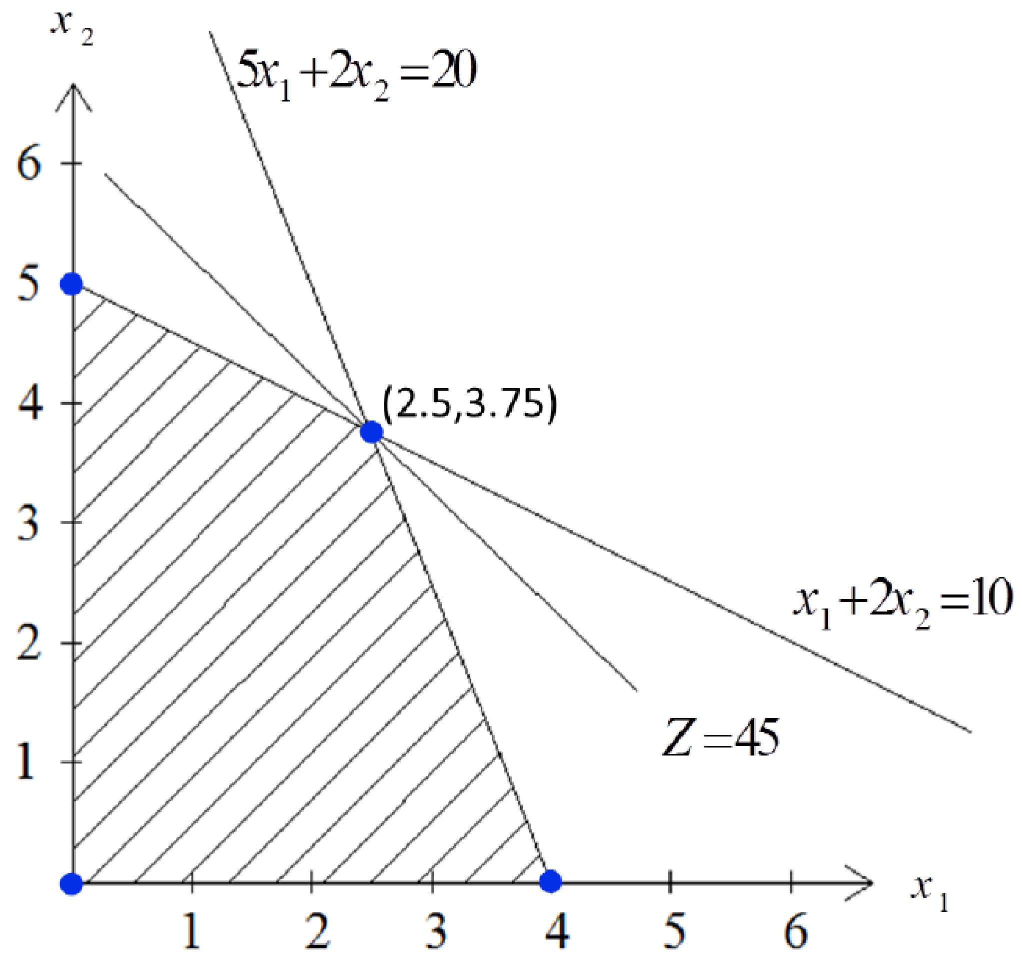
$s.t.$

$$(1) \qquad 5y_1 + y_2 \geq 6$$

$$(2) \qquad 2y_1 + 2y_2 \geq 8$$

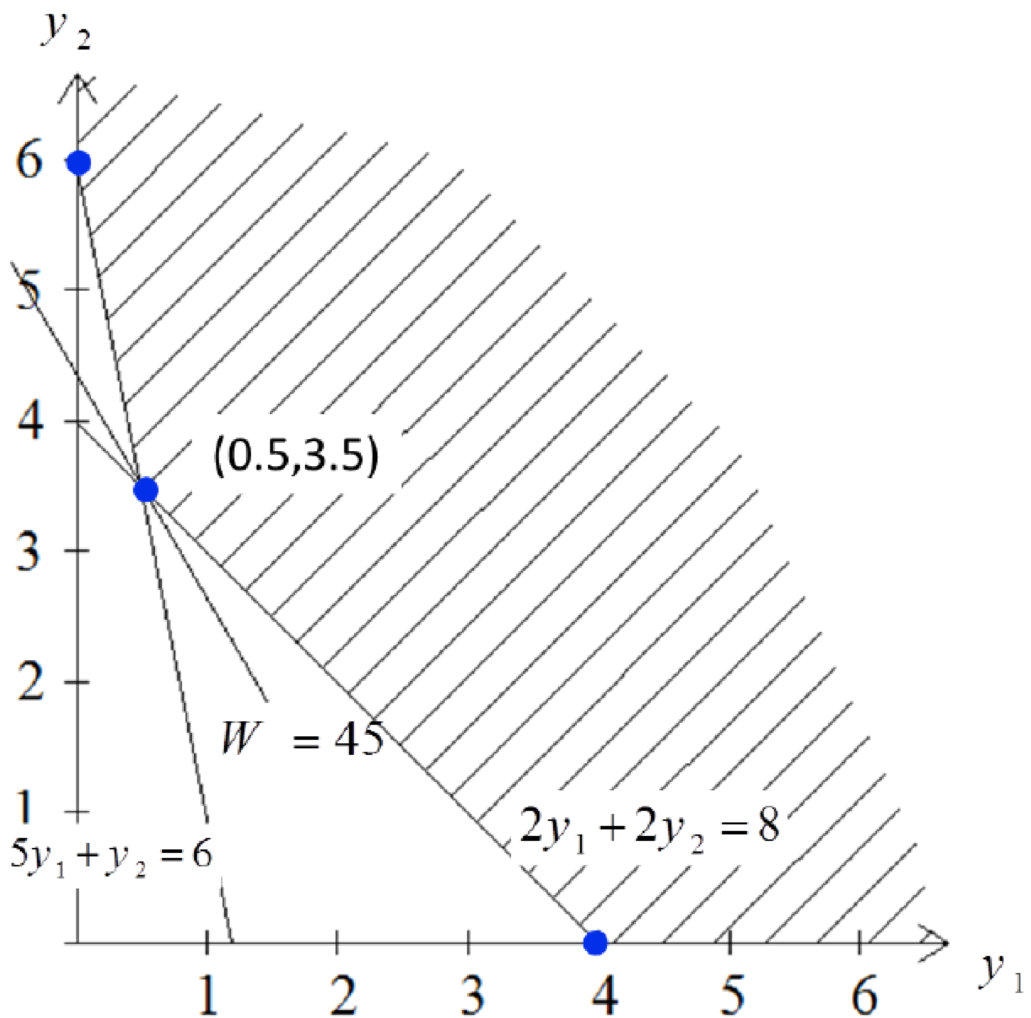
$$y_i \geq 0 \quad i = 1, 2.$$

Relation of primal and dual models



The optimal solution is $(x_1^*, x_2^*) = (2.5, 3.75)$ and $Z^* = 45$.

Relation of primal and dual models



The optimal solution is $(y_1^*, y_2^*) = (0.5, 3.5)$ and $W = 45$.

Relation of primal and dual models

c)

	Primal model			Dual model	
Row	**Basic solution**	**Feasible?**	**Objective function**	**Basic solution**	**Feasible?**
1	(0,5,10,0)	Yes	40	(0,4,-2,0)	No
2	(0,0,20,10)	Yes	0	(0,0,-6,-8)	No
3	(4,0,0,6)	Yes	24	(1.2,0,0,-28/5)	No
4	**(2.5,3.75,0,0)**	**Yes**	**45**	**(0.5,3.5,0,0)**	**Yes**
5	(0,10,0,-10)	No	80	(4,0,14,0)	Yes
6	(10,0,-3,0)	No	60	(0,6,0,4)	Yes

Relation of primal and dual models

d)

Basic variable	Row	Z	X_1	X_2	X_3	X_4	Right Hand Side	
Z	0	1	-6	-8	0	0	0	Non optimal
X_3	1	0	5	2	1	0	20	20/2
X_4	2	0	1	2	0	1	10	Min 10/2

Primal : $(0, 0, 20, 10)$

Dual : $(0, 0, -6, -8)$

Relation of primal and dual models

Basic variable	Row	Z	X_1	X_2	X_3	X_4	Right Hand Side	
Z	0	1	-2	0	0	4	40	Non optimal
X_3	1	0	4	0	1	-1	10	Min 10/4
X_2	2	0	0.5	1	0	0.5	5	5/0.5

Primal : $(0,5,10,0)$

Dual : $(0,4,-2,0)$

Relation of primal and dual models

Basic variable	Row	Z	X_1	X_2	X_3	X_4	Right Hand Side
Z	0	1	0	0	0.5	3.5	45
X_1	1	0	1	0	0.25	-0.25	2.5
X_2	2	0	0	1	-0.13	0.625	3.75

Optimal

Primal : $(2.5, 3.75, 0, 0)$

Dual : $(0.5, 3.5, 0, 0)$

Dual Simplex Method

The dual simplex method can be thought of as the mirror image of the simplex method. The simplex method deals directly with basic solutions in the primal problem that are primal feasible but not primal optimal or not dual feasible. It then moves toward an optimal solution to achieve dual feasibility as well (the optimality test for the simplex method). By contrast, the dual simplex method deals with basic solutions in the primal problem that are dual feasible(primal optimal) but not primal feasible. It then moves toward an optimal solution to achieve primal feasibility as well.

Dual simplex method

In maximization form, the problem to be solved is

$$Max\ Z = \sum_{j=1}^{n} c_j x_j$$

$s.t.$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad i = 1, \dots, m$$

$$x_j \ge 0 \quad j = 1, \dots, n.$$

$$Max\ Z = \sum_{j=1}^{n} c_j x_j$$

$s.t.$

$$\sum_{j=1}^{n} a_{ij} x_j + S_i = b_i \quad i = 1, \dots, m$$

$$x_j \ge 0 \quad j = 1, \dots, n.$$

Dual simplex method

Step 0: After converting any functional constraints in $\geq$ form to $\leq$ form (by multiplying through both sides by -1), introduce slack variables (if needed) to construct a set of equations in equal form. Find a basic solution such that the coefficients in row 0 (objective function) are zero for basic variables and nonnegative for nonbasic variables (so the solution is optimal if it is feasible). Go to the **step 1**.

Step 1: Check to see all the basic variables are nonnegative ($b_i \geq 0 \; \forall i = 1, \ldots, m$). If they are, then this solution is feasible, and therefore optimal, so stop. Otherwise ($b_i < 0 \; \exists i$), go to an **step 2**.

Dual simplex method

Step 2: Follow the steps below:

Step 2-1: Determine the leaving basic variable. Select the negative basic variable,r, that has the largest absolute right hand side, meaning:

$$\bar{b}_r = \underset{i}{Max} \{ |b_i| \mid b_i < 0 \}$$

If all the coefficients of the variables in the r-th row are Non-negative $a_{rj} \geq 0 \quad \forall j = 1, \dots, n$), stop because the problem is infeasible. If $a_{rj} < 0 \, \exists j$ holds, go to step 2-2.

Step 2-2: Determine the entering basic variable. Select the nonbasic variable being made by checking the nonbasic variables with negative coefficients ($a_{rj} < 0$) in that equation and selecting the one with the smallest absolute value of the ratio of the row 0 coefficient to the coefficient in that equation, Meaning:

$$\frac{c_s}{a_{rs}} = \underset{j}{Min} \left\{ \left| \frac{c_j}{a_{rj}} \right| \mid a_{rj} < 0 \right\}$$

Dual simplex method

In above rule, we enter variable s to the basis and leave variable r from basis. Put a box around the row front of variable x_r, and call this the pivot row. Put a box around the column below of variable x_s, and call this the pivot column. Also call the number that is in both boxes the pivot number.

Step 2-3: Determine the new basic solution by using elementary row operations as follows:

1- Divide the pivot row by the pivot number.

2- For each other row (including row 0) that has a negative coefficient in the pivot column, add to this row the product of the absolute value of this coefficient and the new pivot row.

3- For each other row that has a positive coefficient in the pivot column, subtract from this row the product of this coefficient and the new pivot row.

Go to **step 1**.

Dual simplex method

Example: Consider the following problem.

$$Max \quad Z = -4x_1 - 12x_2 - 18x_3$$

$s.t.$

$$\begin{aligned} &(1) \quad x_1 \qquad + 3x_3 \geq 3 \\ &(2) \qquad 2x_2 + 2x_3 \geq 5 \\ &\qquad x_i \geq 0 \quad i = 1, 2, 3. \end{aligned}$$

Use the dual simplex method to solve this problem.

Dual simplex method

Solution:

$$
\begin{aligned}
&Max \quad Z = -4x_1 - 12x_2 - 18x_3 \\
&s.t. \\
&\quad (1) \quad -x_1 \qquad\qquad - 3x_3 + x_4 \qquad = -3 \\
&\quad (2) \qquad\qquad -2x_2 \quad - 2x_3 \qquad + x_5 = -5 \\
&\qquad\qquad x_i \ge 0 \quad i = 1, \ldots, 5.
\end{aligned}
$$

Dual simplex method

The initial basic solution $(0,0,0,-3,-5)$ with $Z=0$ which is not feasible because x_4 and x_5 are the negative values. Dual simplex method is applied in the following table.

Iteration	Basic variable	Row	Z	X_1	X_2	X_3	X_4	X_5	Right Hand Side	
0	Z	0	1	4	12	18	0	0	0	Non optimal
	X_4	1	0	-1	0	-3	1	0	-3	
	X_5	2	0	0	-2	-2	0	1	-5	
1	Z	0	1	4	0	6	0	6	-30	Non optimal
	X_4	1	0	-1	0	-3	1	0	-3	
	X_2	2	0	0	1	1	0	-0.5	2.5	
2	Z	0	1	2	0	0	2	6	-36	Optimal
	X_3	1	0	0.33	0	1	-0.33	0	1	
	X_2	2	0	-0.33	1	0	0.33	-0.5	1.5	

Dual simplex method

Example: Consider the following model:

$$Max \quad Z = -5x_1 - 10x_2$$

$$s.t.$$

$$\begin{array}{lrl} (1) & 2x_1 + x_2 & \leq 40 \\ (2) & x_2 & \geq 15 \\ (3) & -2x_1 + x_2 & \leq 10 \\ & x_i \geq 0 \quad i = 1,2. & \end{array}$$

a) Solve the problem graphically.

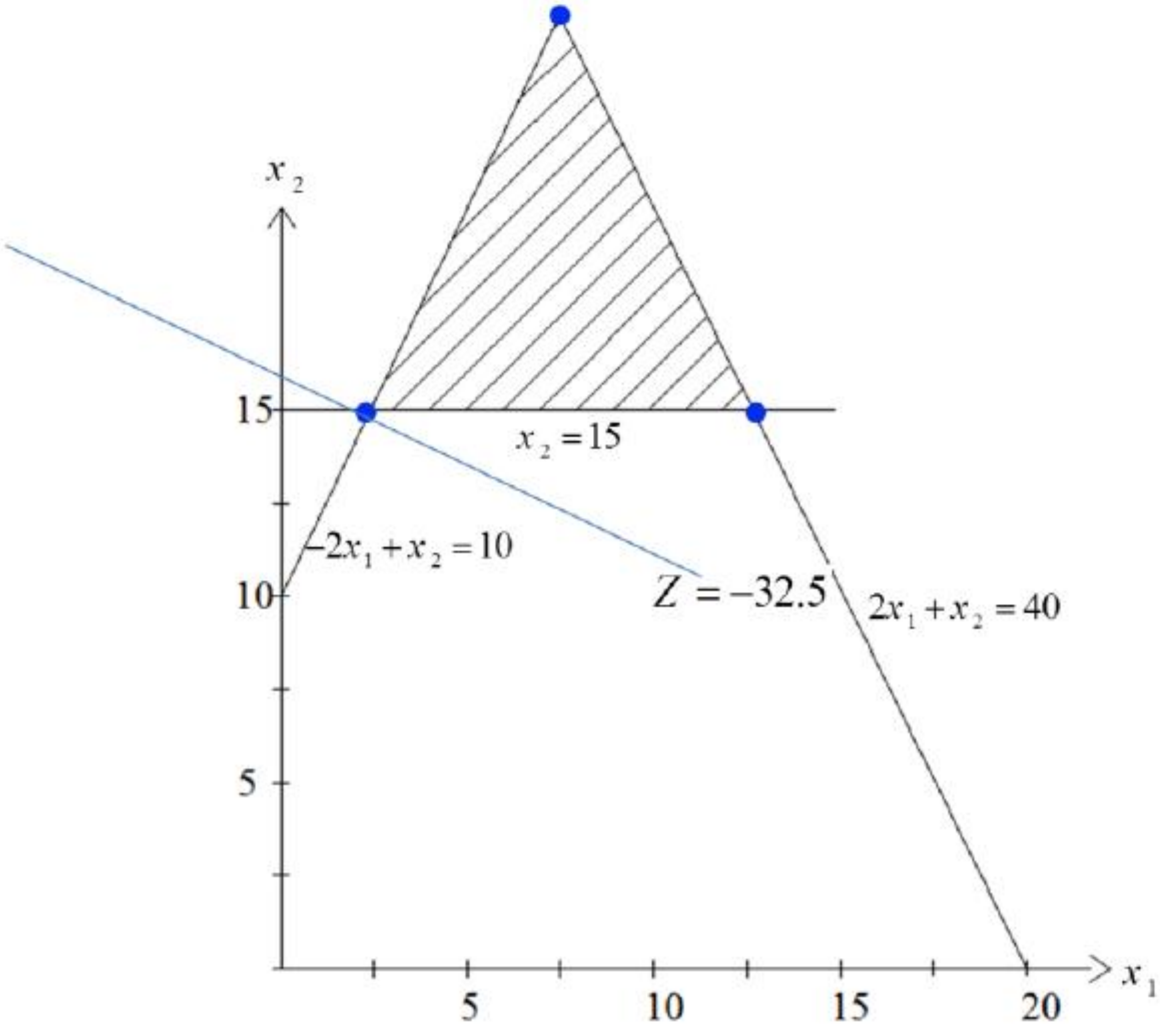
b) Use the dual simplex method manually to solve this problem.

c) Draw the path connecting the basic solutions.

Dual simplex method

Solution:

a)



Dual simplex method

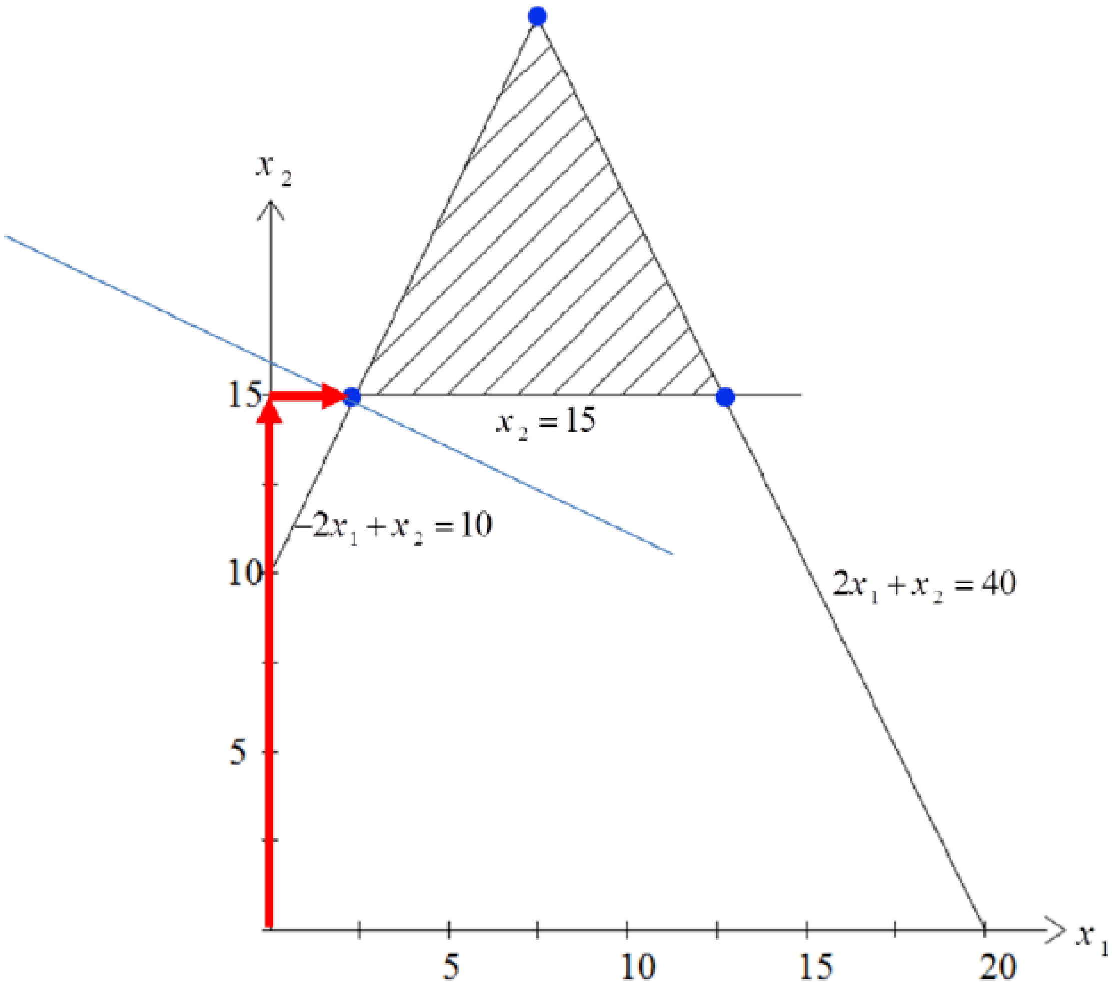
b)

Basic solution	Row	Z	X_1	X_2	X_3	X_4	X_5	RHS	
Z	0	1	1	2	0	0	0	0	Non optimal
X_3	1	0	2	1	1	0	0	40	
X_4	2	0	0	-1	0	1	0	-15	
X_5	3	0	-2	1	0	0	1	10	
Z	0	1	2	0	0	2	0	-30	Non-optimal
X_3	1	0	0	0	1	1	0	25	
X_2	2	0	0	1	0	-1	0	15	
X_5	3	0	-2	0	0	1	1	-5	
Z	0	1	0	0	0	2.5	0.5	-32.5	Optimal
X_3	1	0	0	0	1	2	1	20	
X_2	2	0	0	1	0	-1	0	15	
X_1	3	0	1	0	0	-0.5	-0.5	2.5	

Dual simplex method

c) we draw the path of basic solutions for the dual model

$$(0,0) \rightarrow (0,15) \rightarrow (2.5,15)$$



Prime-dual simplex algorithm

The prime-dual simplex algorithm

This algorithm is used for problems that are infeasible (due to the presence of negative numbers on the right hand side of the constraints) and non-optimal (due to the presence of negative numbers in the row 0). The advantage of this algorithm compared to the previous methods is that there is no need to add an artificial variable. The prime-dual algorithm is based on the concepts of the prime simplex and the dual simplex algorithm.

Note:

Prime-dual simplex algorithm

Steps of the prime-dual simplex algorithm

Step 1. Like the standard form of the simplex method, transform all the constraints of the problem into a smaller or equal form and the objective function into a maximum.

Step 2. After adding auxiliary variables to the constraints, enter the problem into the simplex table.

Step 3. Calculate the amount of change in the objective function by calculating the effect of using the prime simplex or the dual simplex as follows:

A) prime simplex effect: This effect is checked when there is a variable with a **negative** coefficient in the row 0 and the number of the pivot column and the value on the right hand side opposite this number are non-negative. This effect is calculated as follows.

$$\left|\frac{b_i \times c_j}{a_{ij}}\right|$$

Prime-dual simplex algorithm

b) Dual simplex effect: This effect can be calculated when there is a variable in the zero row with a **non-negative** coefficient and the number of the pivot column and the value on the right hand side opposite this number are negative. This effect is similar to part A and is calculated as follows.

$$\left|\frac{b_i \times c_j}{a_{ij}}\right|$$

Step 4. We select the largest calculated absolute value related to the effect of the prime and dual simplex and act according to the prime or dual algorithms. If it is not possible to use a prime or a dual simplex, you have reached the end of the operation, otherwise go to step 3.

Prime-dual simplex algorithm

Example: Consider the following problem.

$$
\begin{aligned}
Max\ \ Z &= -3x_1 + 6x_2 \\
& x_1 + 2x_2 \geq 6 \\
& 3x_1 + x_2 \geq 9 \\
& 7x_1 + 5x_2 \leq 35 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Solution:

Based on steps 1 and 2, we transform the problem as follows.

$$
\begin{aligned}
Max\ \ Z &= -3x_1 + 6x_2 \\
& -x_1 - 2x_2 + s_1 = -6 \\
& -3x_1 - x_2 + s_2 = -9 \\
& 7x_1 + 5x_2 + s_3 = 35 \\
& x_1, x_2, s_1, s_2, s_3 \geq 0
\end{aligned}
$$

Prime-dual simplex algorithm

Prime simplex effect = $=\left|\dfrac{35\times 6}{5}\right|=42$

Prime-dual simplex algorithm

Dual simplex effect = $= \left| \dfrac{(-6)\times 3}{(-1)} \right| = 18$

Dual simplex effect = $= \left| \dfrac{(-9)\times 3}{(-3)} \right| = 9$

Prime-dual simplex algorithm

Pivot column – Prime simplex

Table 1

Basic variable	Row	Z	x_1	x_2	s_1	s_2	s_3	RHS
Z	0	1	3	-6	0	0	0	0
s_1	1	0	-1	-2	1	0	0	-6
s_2	2	0	-3	-1	0	1	0	-9
s_3	3	0	7	5	0	0	1	35

Pivot row– Prime simplex

Prime-dual simplex algorithm

In Table 2, since there is no negative number in the zero row, it is not possible to calculate the initial simplex effect. But since the only negative number related to the basic variable s_2 is -2, the only negative number is -1.6 in row 2, and $\frac{57}{5} > 0$, it is possible to calculate the dual simplex effect. This effect, which is also the only effect, is calculated as follows:

$$= \left| \frac{(-2)\times(\frac{57}{5})}{(-\frac{8}{5})} \right| = 14.25$$

Prime-dual simplex algorithm

Pivot column – Dual simplex

Table 2

Basic variable	Row	Z	x_1	x_2	s_1	s_2	s_3	RHS
Z	0	1	$\frac{57}{5}$	0	0	0	$\frac{6}{5}$	42
s_1	1	0	$\frac{9}{5}$	0	1	0	$\frac{2}{5}$	8
s_2	2	0	$-\frac{8}{5}$	0	0	1	$\frac{1}{5}$	-2
x_2	3	0	$\frac{7}{5}$	1	0	0	$\frac{1}{5}$	7

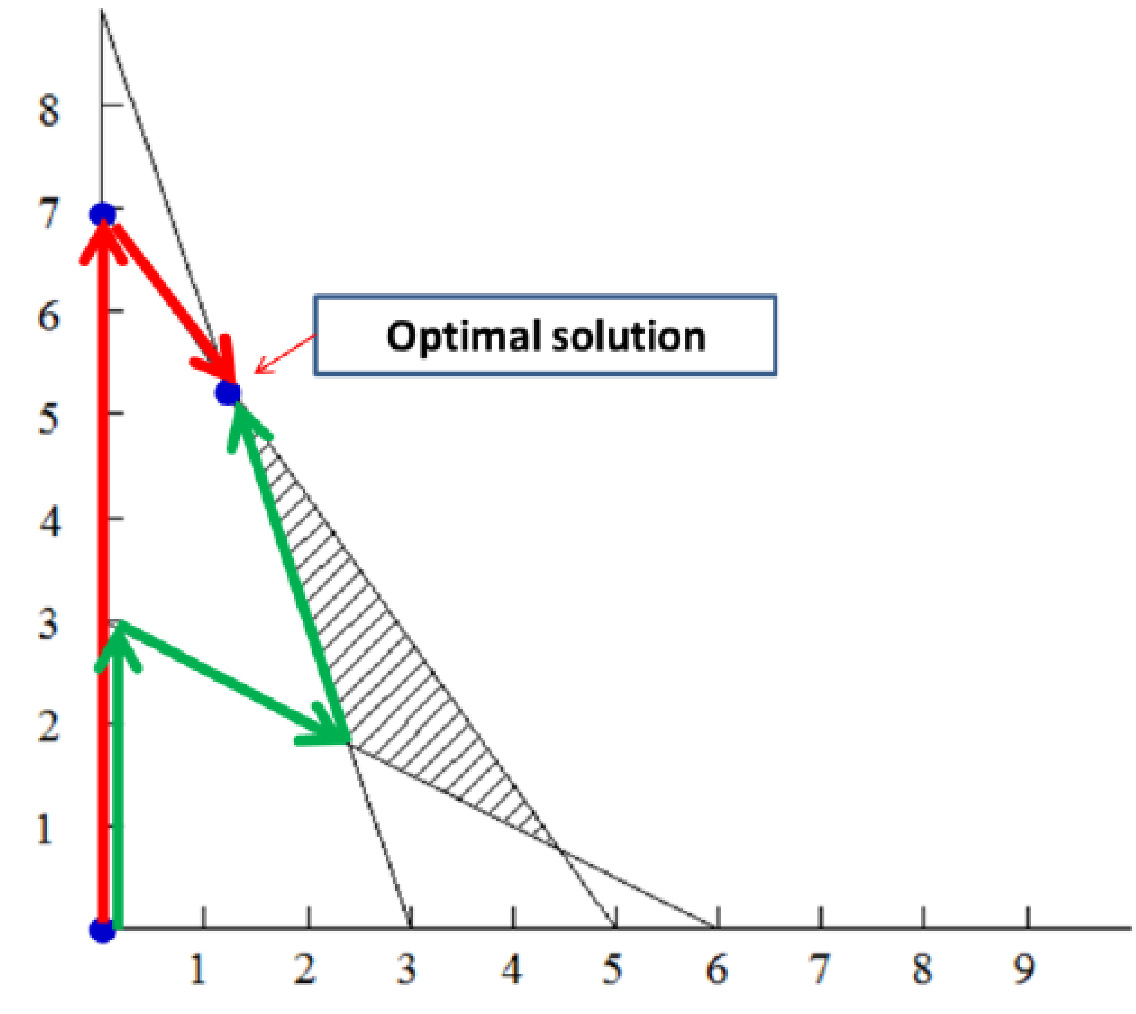
Pivot row– Dual simplex

Prime-dual simplex algorithm

Table 3

Basic variable	Row	Z	x_1	x_2	s_1	s_2	s_3	RHS
Z	0	1	0	0	0	$\frac{57}{8}$	$\frac{21}{8}$	$\frac{111}{4}$
s_1	1	0	0	0	1	$\frac{9}{8}$	$\frac{5}{8}$	$\frac{23}{4}$
x_1	2	0	1	0	0	$-\frac{5}{8}$	$-\frac{1}{8}$	$\frac{5}{4}$
x_2	3	0	0	1	0	$\frac{7}{8}$	$\frac{3}{8}$	$\frac{21}{4}$

Prime-dual simplex algorithm



Important theorems in duality

Two important theorems in dual theory

Weak duality theorem

if $(x_1,x_2,\ldots,x_n)$ is a feasible solution of the prime problem and $(y_1,y_2,\ldots,y_m)$ is a feasible solution of the dual problem, then we have:

$$\sum_{j=1}^{n} c_j x_j \leq \sum_{i=1}^{m} b_i y_i$$

Important theorems in duality

Strong duality theorem

if the prime problem has an optimal solution (x_1,x_2,…,xn), its dual problem also has an optimal solution (y_1,y_2,…,y_m) so that:

$$\sum_{j=1}^{n} c_j x_j = \sum_{i=1}^{m} b_i y_i$$

Exercises

Thanks

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