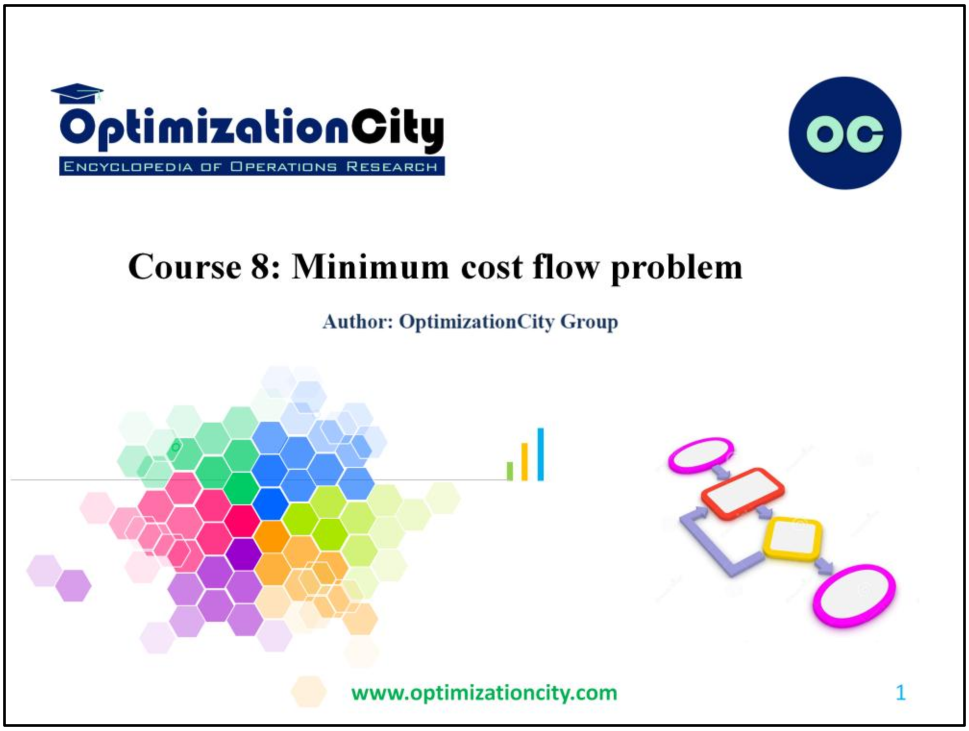


Welcome to the eighth course on our website, where we will explore the concept of Minimum cost flow problem! The minimum cost flow problem is a puzzle in network optimization that asks for the cheapest way to route a given amount of flow through a network, considering capacity limits and aiming to minimize the total cost involved.

فهرست مطالب

In this course, we will cover important topics related to graph optimization. The minimum cost flow problem is like a puzzle where we aim to find the most affordable way to move things through a network. Then We use special methods to solve this puzzle and figure out the cheapest route. Afther, To start, we make an initial plan considering things like how much can be carried and wanting to spend the least amount of money. Finally, By doing exercises and practice activities, we can get better at understanding and solving this problem.

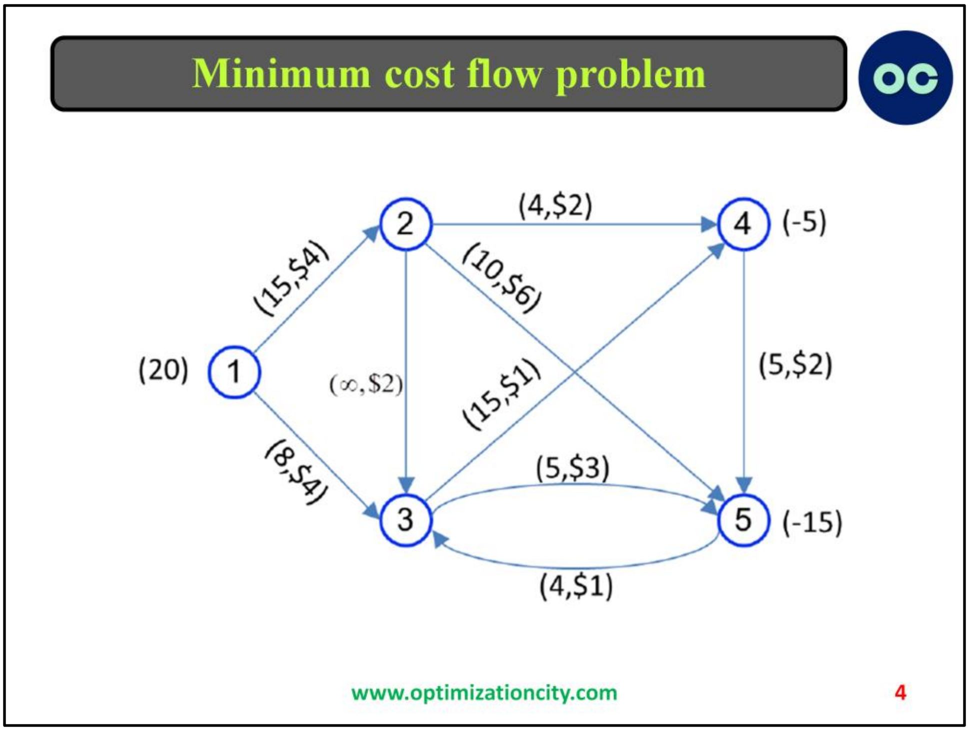
Minimum cost flow problem

OC

Minumum cost flow problem

In the problem of minimum cost flow, we are looking for homogeneous product distribution from the factory (origin) to the sales market (destination). Suppose the number of products manufactored in each factory and the number of required products are known. Also, it is not necessary to send the products directly to the destinations, but it is possible to send it to the distribution centers through an intermediate point. In addition, transportation lines are limited in terms of line capacity. The goal in this issue is to minimize the cost of transporting products.

In the minimum cost flow problem, we want to distribute the same type of product from the factory to the sales market efficiently. We know the quantity of products produced at each factory and the required quantity at each destination. Instead of sending products directly to the destinations, we can send them to distribution centers through an intermediate point if needed. However, there are limits on how much can be transported on each transportation line. The objective is to minimize the cost of transporting the products while meeting the demand.



Let's look at a simple example of the minimum cost flow problem using the diagram below. The nodes are shown as numbered circles, and the arcs are represented by arrows. The arcs have a specific direction, meaning materials or goods can be transported from one node to another in that direction. For instance, we can send materials from node 1 to node 2, but we cannot send them from node 2 to node 1. We use the notation i-j to indicate the arc going from node i to node j.

In the diagram above, each arrow represents a path with a specific capacity and cost for transportation. The capacity indicates the maximum amount that can be transported along that path, while the cost per unit represents the expense associated with each unit of transportation. For instance, in the arrow (2-4), the flow can range from 0 to 4 units, and each unit passing through this path costs $2. The symbol ∞ indicates that there is no limit on the capacity of that particular path.

Moreover, the numbers in parentheses next to the circles represent the available supply and the required demand at each node. In this diagram, node 1 is the starting point with a supply of 20 units. Nodes 4 and 5 are the destinations that require 5 and 15 units respectively, indicated by the negative sign.

Minimum cost flow problem

x_{ij}: is the number of units transported from node i to node j using arc i-j.

$$
\begin{array}{l}
Min \quad 4x_{12} + 4x_{13} + 2x_{23} + 2x_{24} + 6x_{25} + x_{34} + 3x_{35} + 2x_{45} + x_{53} \\
s.t. \\
\begin{array}{lllllllllll}
(1) & x_{12} & +x_{13} & & & & & & & & = 20 \\
(2) & -x_{12} & & +x_{23} & +x_{24} & +x_{25} & & & & & = 0 \\
(3) & & -x_{13} & -x_{23} & & & +x_{34} & +x_{35} & & -x_{53} & = 0 \\
(4) & & & & -x_{24} & & -x_{34} & & +x_{45} & & = -5 \\
(5) & & & & & -x_{25} & & -x_{35} & -x_{45} & +x_{53} & = -15
\end{array} \\
x_{12} \le 15; x_{13} \le 8; x_{23} \le \infty; x_{24} \le 4; x_{25} \le 10; x_{34} \le 15; x_{35} \le 5; x_{45} \le \infty; x_{53} \le 4.
\end{array}
$$

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In the minimum cost flow problem, our objective is to discover the flow pattern that incurs the lowest cost. To formulate this problem as a linear programming model, let's consider the following notation:
xij: represents the quantity of units transported from node i to node j using the arc i-j.
Now, let's present the linear programming model for the minimum cost flow problem.

Equations 1 to 5 represent the flow balance equations in the network. Let's consider the balance equation at node 1 as an example.
The equation states that the combined flow leaving node 1 (x12 + x13) should be equal to the supply available at node 1, which is 20 units. This ensures that the outgoing flow matches the available supply.

Similarly, the balance equation at node 2 indicates that the incoming flow to node 2 (x12) is equal to the outgoing flow from node 2 (x23 + x24 + x25). This ensures that the flow entering and leaving node 2 is properly balanced.

The minimum cost flow model in the network has a unique structure that helps determine the solution algorithm. The flow variables, denoted as xij in the balance equations, have coefficients of 0, +1, or -1. Additionally, these variables appear precisely in two balance equations: once with a coefficient of +1 representing the node where the flow originates, and once with a coefficient of -1 representing the

node where the flow enters.
Based on the above characteristics, the general form of the Minimum Cost Flow problem can be expressed as follows.

Minimum cost flow problem

$$Min \quad \sum_i \sum_j c_{ij} x_{ij}$$

$s.t.$

$$\sum_j x_{ij} - \sum_k x_{ki} = b_i \quad i = 1, \dots, n$$

$$l_{ij} \le x_{ij} \le u_{ij}$$

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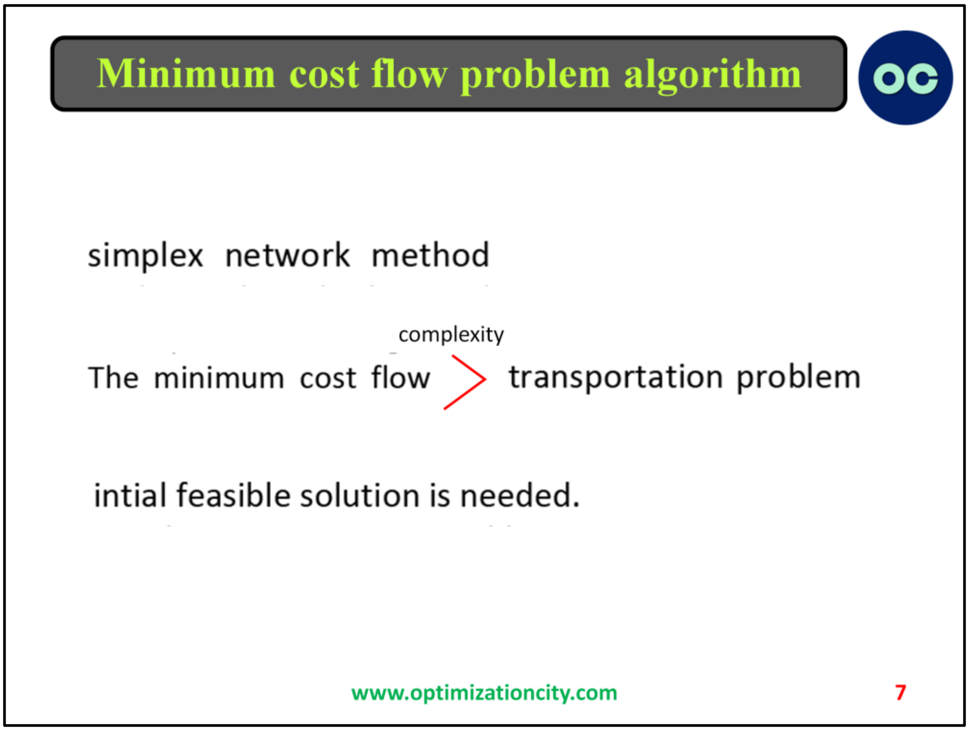
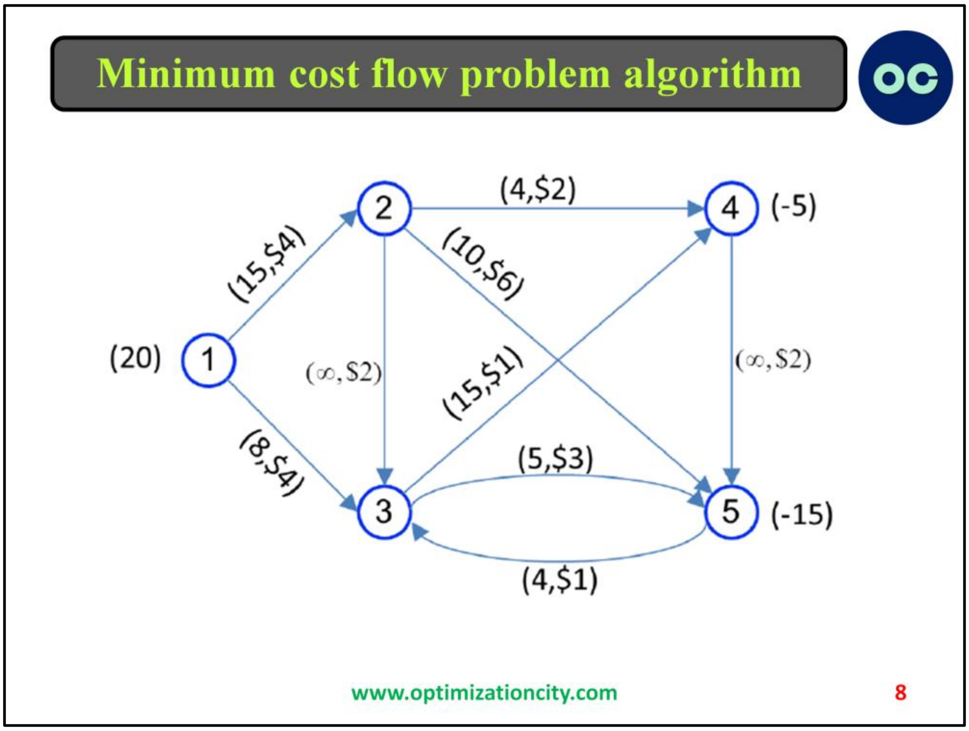
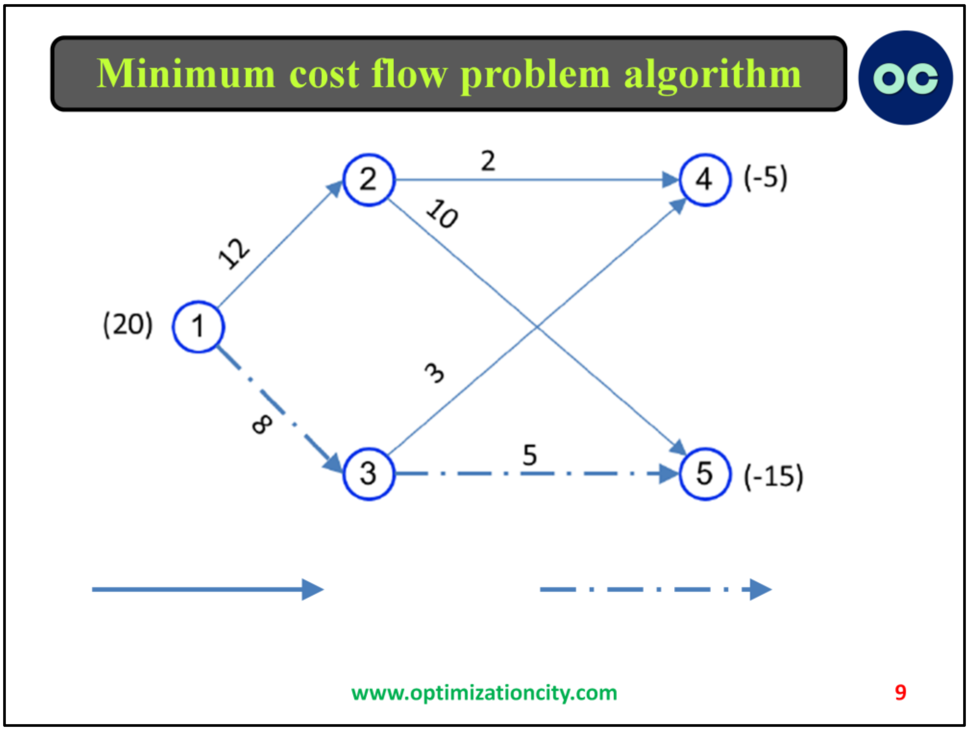
The model shown here represents the general form of the minimum cost flow problem. However, we can simplify this model into more straightforward forms, which are explained below.



In this part, we will learn about the simplex network method, which builds upon the network concepts discussed earlier. The minimum cost flow problem is more complex than the transportation problem because it involves intermediate nodes and capacity restrictions on each connection. To solve this problem using the simplex method, we need to find an initial feasible solution. This process is slightly different from the transportation problem and requires significant effort. Let's assume that we already have an initial feasible solution. We will illustrate the application of the simplex method to the minimum cost flow problem through an example.



In the diagram, each arc is labeled with two values: A and B. A represents the capacity of the arc, indicating how much can pass through it, while B represents the cost of moving one unit through the arc. Now let's take a look at the initial solution that we will consider.



In the picture above, the arcs represented by .._ are non-basic arcs that have reached their maximum capacity, while the solid arcs (represented by ___) are basic arcs. These solid arcs form a spanning tree, which serves as the initial solution. To determine if this solution is optimal, we need to calculate the cost of all the non-basic arcs. To do this, we calculate yi values for each arc (where i ranges from 1 to n). If the calculated yi values satisfy certain conditions, we can evaluate the optimality of the solution.

Minimum cost flow problem algorithm

If the y_i applies in the following relations:

$$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$$

$$c_{ij} - y_i + y_j = 0.$$

If the calculated yi values meet certain conditions, then the current basic solution is considered optimal. To calculate the yi values, we can arbitrarily set one of the yi values to zero and determine the remaining yi values using the formula cij - yi + yj = 0. Let's compute the yi values based on this equation.

Minimum cost flow problem algorithm

$$y_2 = 0 \rightarrow \begin{cases} (2,4) \rightarrow 0 = 2 - y_2 + y_4 \rightarrow y_4 = -2 \\ (1,2) \rightarrow 0 = 4 - y_1 + y_2 \rightarrow y_1 = +4 \\ (3,4) \rightarrow 0 = 1 - y_3 + y_4 \rightarrow y_3 = -1 \\ (2,5) \rightarrow 0 = 6 - y_2 + y_5 \rightarrow y_5 = -6 \end{cases}$$

The cost of $\overline{c_{ij}}$ for non-basic variables is as follows:

$$\overline{c_{13}} = 4 - 4 + (-1) = -1$$
$$\overline{c_{23}} = 2 - 0 + (-1) = +1$$
$$\overline{c_{35}} = 3 - (-1) + (-6) = -2$$
$$\overline{c_{45}} = 2 - (-2) + (-6) = -2$$
$$\overline{c_{53}} = 1 - (-6) + (-1) = +6$$

The cost associated with non-basic variables ij which is cijbar is as follows:

Minimum cost flow problem algorithm

To improve the current solution based on the above $\overline{c_{ij}}$, we have two solutions:

1- Increasing the arc flow that has a negative $\overline{c_{ij}}$ and is now at its lower bound.

2- Reducing the arc flow that has a positive $\overline{c_{ij}}$ and is now at its upper bound.

$$\overline{c_{13}} = 4 - 4 + (-1) = -1$$
$$\overline{c_{23}} = 2 - 0 + (-1) = +1$$
$$\overline{c_{35}} = 3 - (-1) + (-6) = -2$$
$$\overline{c_{45}} = 2 - (-2) + (-6) = -2$$
$$\overline{c_{53}} = 1 - (-6) + (-1) = +6$$

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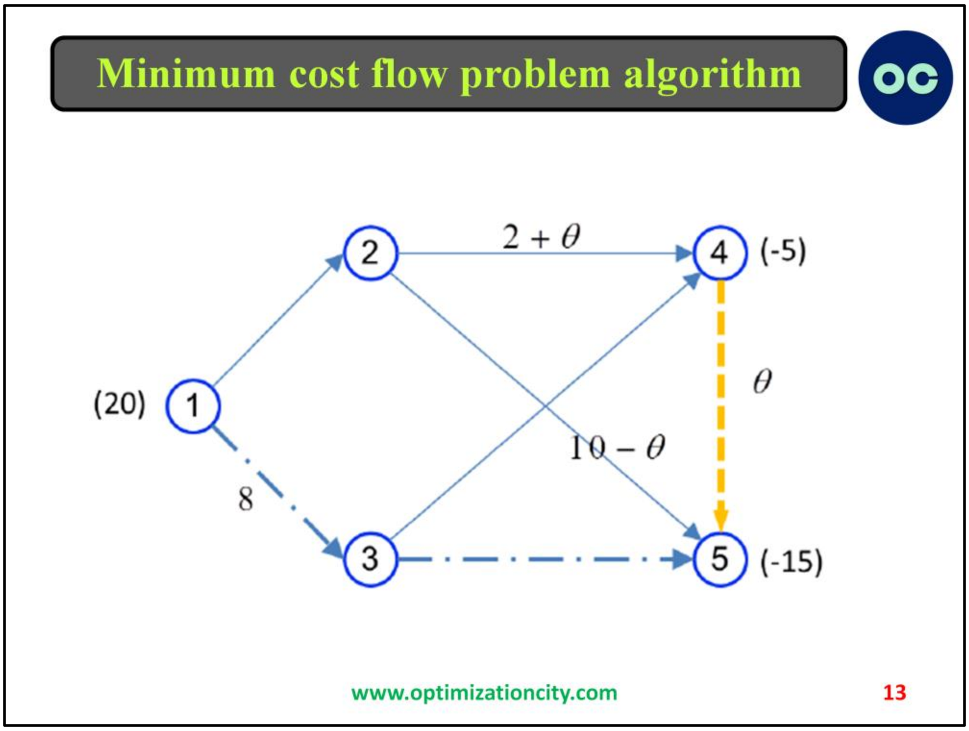
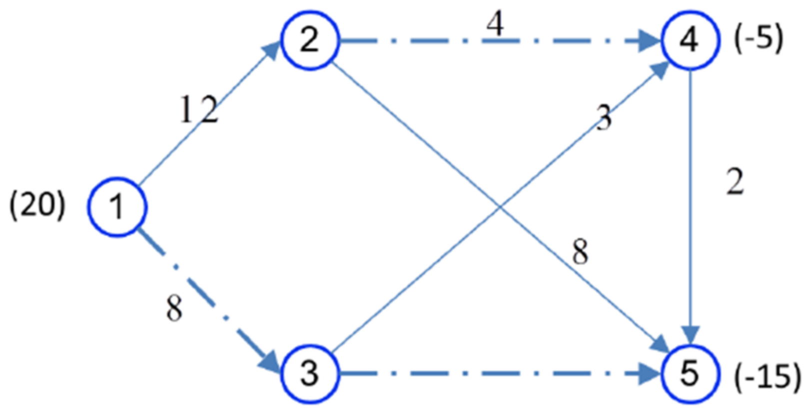
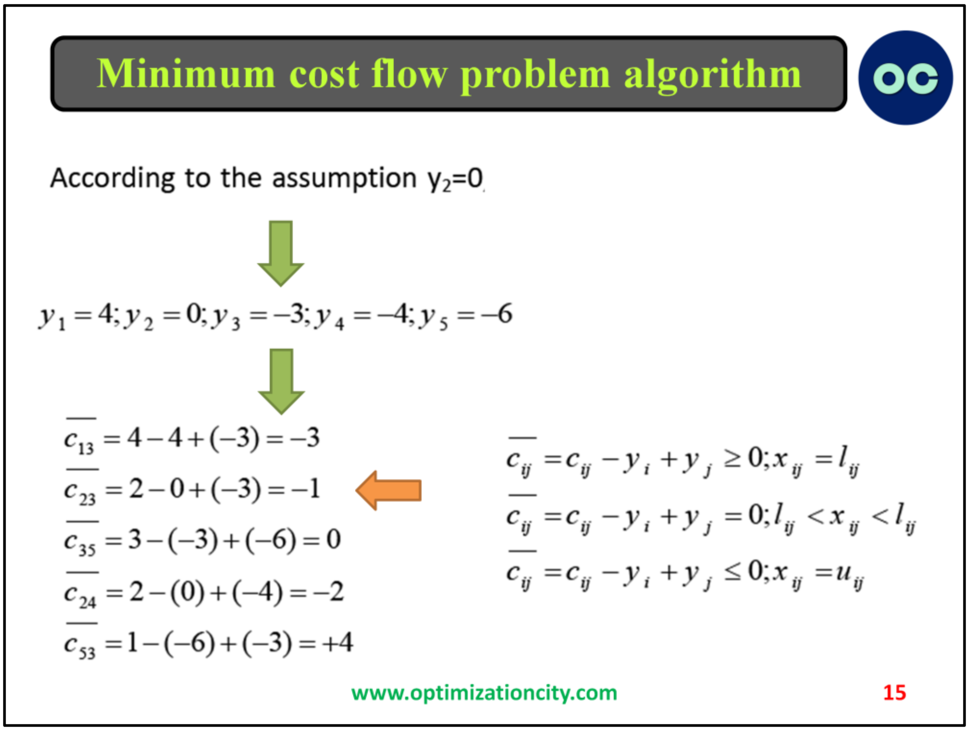
To improve the current solution using the provided cij values, we have two options:
Increase the flow through an arc that has a negative cij and is currently at its minimum limit.
Decrease the flow through an arc that has a positive cij and is currently at its maximum limit.
Considering these cases, the only possible candidate is the arc (4,5). In the following diagram, we added the arc (4,5) to the network to determine which arc should be removed from the basic solution.



We can increase the value of θ up to 2, which would cause the arc (2,4) to reach its maximum capacity and be removed from the basic solution. Consequently, the current basic solution is as follows:

Minimum cost flow problem algorithm

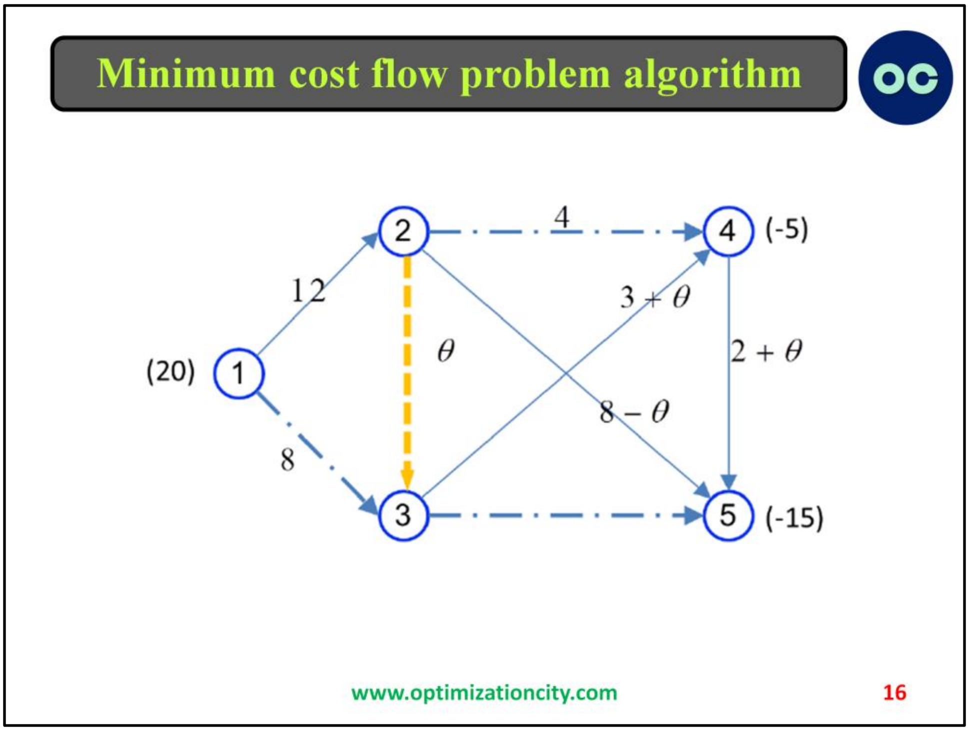




Based on the assumption that y2 is equal to zero, we can calculate the values of yi for the basic variables using the equations cij - yi + yj = 0.

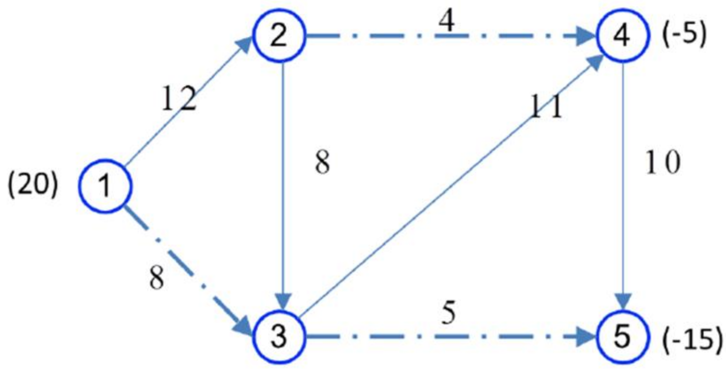
The costs cijbar for identifying the input arc to the basic solution and the output arc from the basic solution are as follows:

At this point, we include the arc (2,3) in the basic solution. To determine the output arc, we proceed as follows:



We can increase the value of θ up to 8, resulting in the removal of the arc (2,5) from the basic solution. As a result, the updated basic solution is as follows:

Minimum cost flow problem algorithm



Minimum cost flow problem algorithm

$$y_1 = 4; y_2 = 0; y_3 = -2; y_4 = -3; y_5 = -5$$

According to the y_is, the amount of $\overline{c_{ij}}$ is as follows:

$$\overline{c_{13}} = 4 - 4 + (-2) = -2$$
$$\overline{c_{25}} = 6 - 0 + (-5) = +1$$
$$\overline{c_{35}} = 3 - (-2) + (-5) = 0$$
$$\overline{c_{24}} = 2 - (0) + (-3) = -1$$
$$\overline{c_{53}} = 1 - (-5) + (-2) = +4$$

$$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$$

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Using the basic solution, we calculate the values of yi.

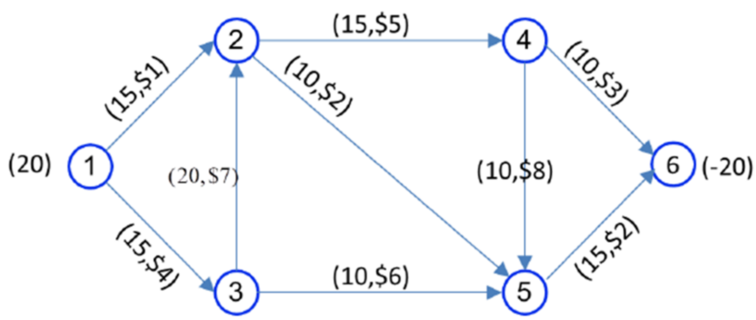
Based on these yi values, we determine the amounts of cijbar as follows:

For non-basic variables, the cijbar values are negative, and the arc flow is at its maximum limit. For basic variables, the cijbar values are positive, and the arc flow is at its minimum limit. Therefore, the current solution is the optimal solution.

Minimum cost flow problem algorithm

Example:

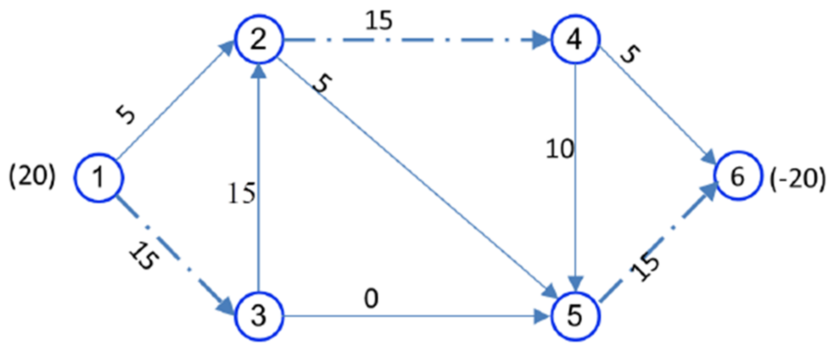
Solve the minimum cost flow problem in the following network using the simplex method. Use the given tree for the initial solution.



Let's solve the minimum cost flow problem in the network shown using the simplex method. We will utilize the provided tree as the initial solution.

Minimum cost flow problem algorithm

Initial solution:



Minimum cost flow problem algorithm

Solution:

$$\overline{c_{12}} = 1 - y_1 + y_2 = 0 \rightarrow y_1 = 1$$
$$\overline{c_{32}} = 7 - y_3 + y_2 = 0 \rightarrow y_3 = 7$$
$$\overline{c_{25}} = 2 - y_2 + y_5 = 0 \rightarrow y_5 = -2$$
$$\overline{c_{45}} = 8 - y_4 + y_5 = 0 \rightarrow y_4 = 6$$
$$\overline{c_{46}} = 3 - y_4 + y_6 = 0 \rightarrow y_6 = 3$$

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Solution:
The value of the objective function for the initial solution is 380. Assuming y2 is equal to zero, we can calculate the yi values using the equation cij - yi + yj = 0.

Minimum cost flow problem algorithm

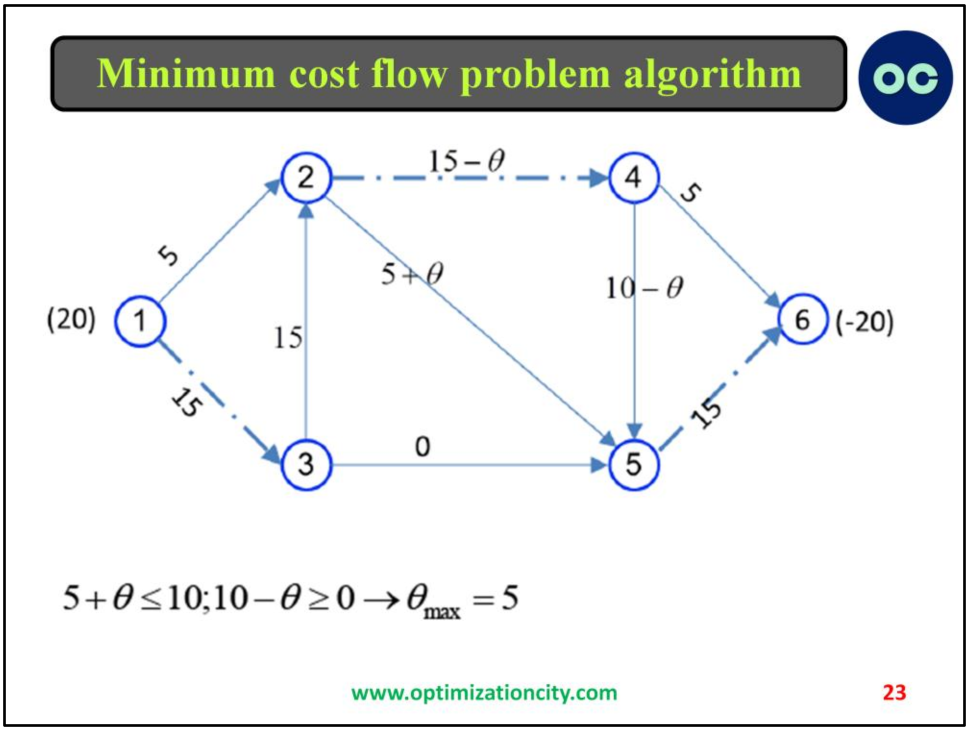
The $\overline{c_{ij}}$ for non-base arcs are as follows.

$$\overline{c_{13}} = 4 - y_1 + y_3 = 4 - 1 + 7 = 10$$
$$\overline{c_{35}} = 6 - y_3 + y_5 = 6 - 7 + (-2) = -3$$
$$\overline{c_{24}} = 5 - y_2 + y_4 = 5 - 0 + 6 = 11$$
$$\overline{c_{56}} = 2 - y_5 + y_6 = 2 + 2 + 3 = 7$$

$$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$$

The costs cijbar for the non-basic arcs are as follows.

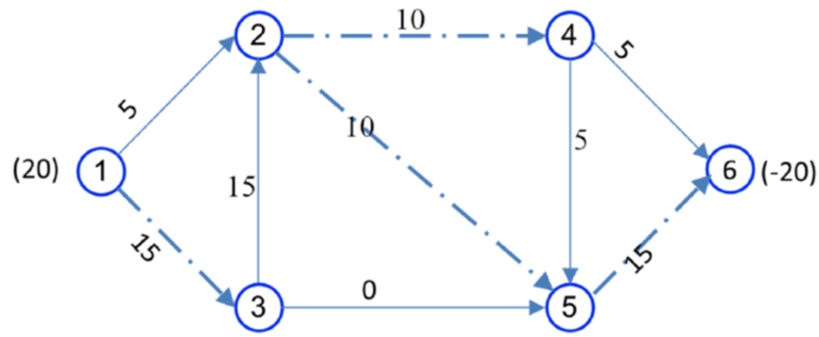
Out of the four arcs that can enter the basic solution, we select the arc (2,4). The output arc is determined as follows.



As the arc (2,5) has reached its maximum capacity, it is removed from the basic solution.

Therefore, the updated basic solution is as follows.

Minimum cost flow problem algorithm



Minimum cost flow problem algorithm

In this solution, the value of the objective function is equal to 295.

$$\overline{c_{12}} = 1 - y_1 + y_2 = 0 \rightarrow y_1 = 1$$
$$\overline{c_{32}} = 7 - y_3 + y_2 = 0 \rightarrow y_3 = 7$$
$$\overline{c_{24}} = 5 - y_2 + y_4 = 0 \rightarrow y_4 = -5$$
$$\overline{c_{45}} = 8 - y_4 + y_5 = 0 \rightarrow y_5 = -13$$
$$\overline{c_{46}} = 3 - y_4 + y_6 = 0 \rightarrow y_6 = -8$$

In this solution, the objective function has a value of 295. Assuming y2 is zero, we can calculate the yi values for the basic arcs using the equations cij - yi + yj = 0 as follows.

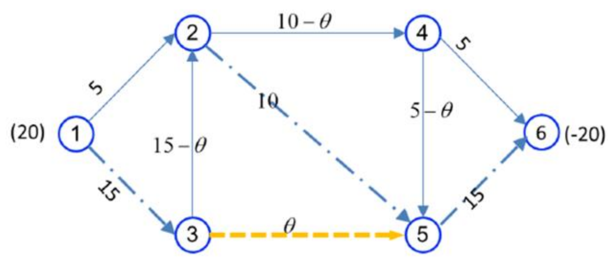
Minimum cost flow problem algorithm

The $\overline{c_{ij}}$ for non-base arcs is as follows.

$$\overline{c_{13}} = 4 - y_1 + y_3 = 4 - 1 + 7 = 10$$
$$\overline{c_{35}} = 6 - y_3 + y_5 = 6 - 7 + (-13) = -14$$
$$\overline{c_{25}} = 2 - y_2 + y_5 = 2 - 0 - 13 = -11$$
$$\overline{c_{56}} = 2 - y_5 + y_6 = 2 + 13 - 8 = 7$$

$$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$$

The arc (3,5) can enter the basic solutions.



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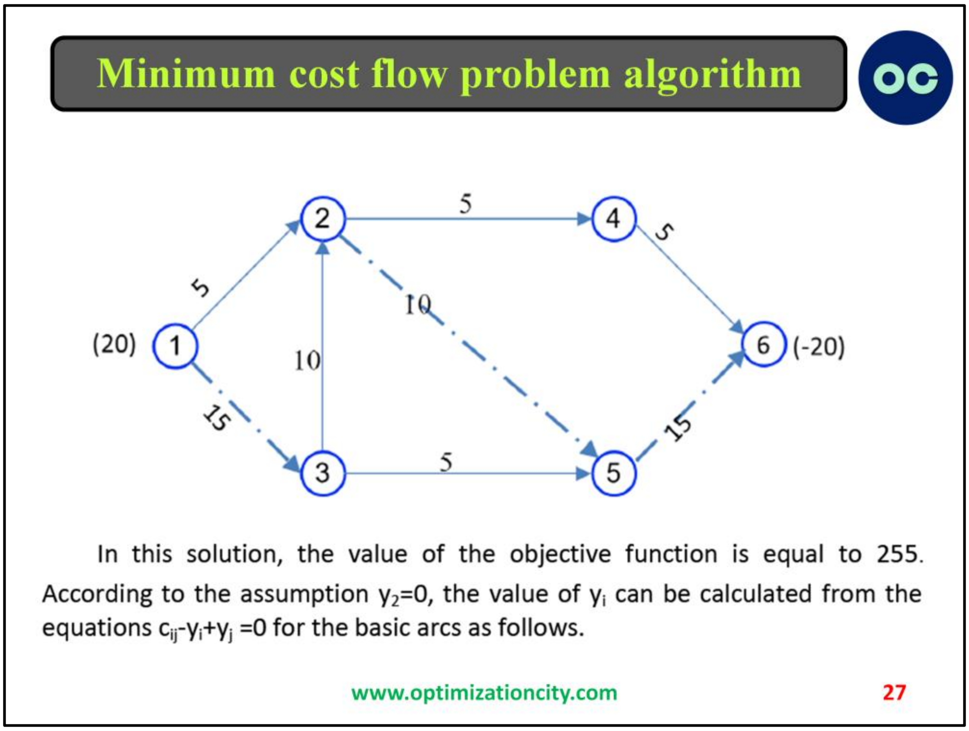
The cijbar cost for the non-basic arcs are as follows.

The arc (3,5) can be included in the basic solutions.



By setting θ to 5, the arc (4,5) is removed from the base, and the arc (3,5) is added to the base. As a result of this change, the current solution is as follows.

In this solution, the objective function has a value of 255. Assuming y2 is zero, we can calculate the yi values for the basic arcs using the equations cij - yi + yj = 0 as follows.

Minimum cost flow problem algorithm

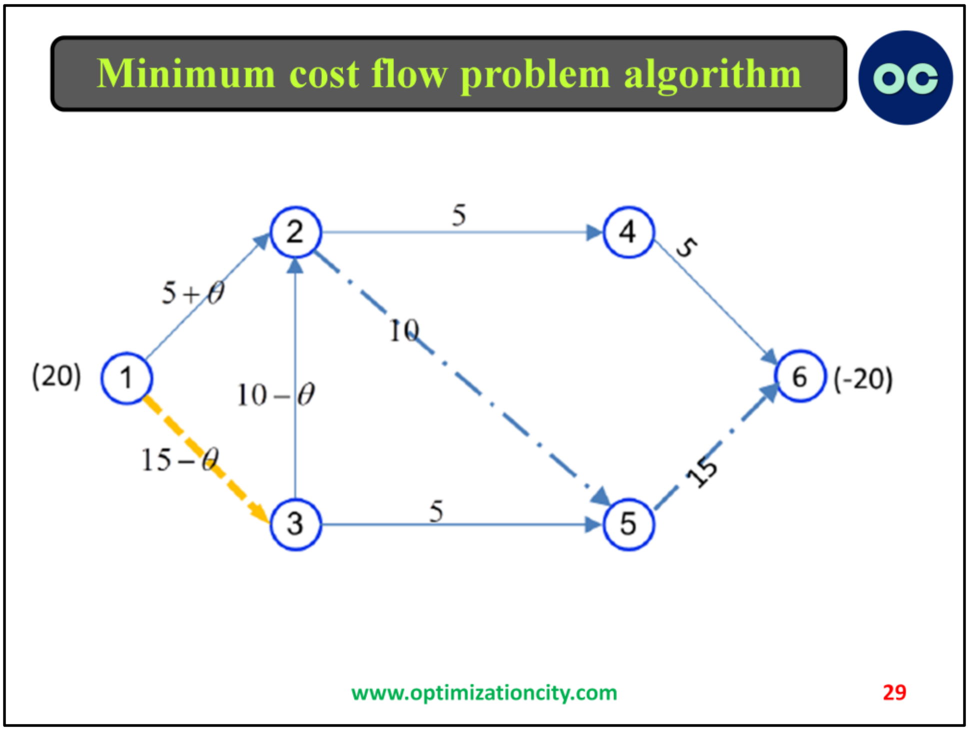
$$\overline{c_{12}} = 1 - y_1 + y_2 = 0 \rightarrow y_1 = 1$$
$$\overline{c_{32}} = 7 - y_3 + y_2 = 0 \rightarrow y_3 = 7$$
$$\overline{c_{24}} = 5 - y_2 + y_4 = 0 \rightarrow y_4 = -5$$
$$\overline{c_{46}} = 3 - y_4 + y_6 = 0 \rightarrow y_6 = -8$$
$$\overline{c_{35}} = 6 - y_3 + y_5 = 0 \rightarrow y_5 = 1$$

The $\overline{c_{ij}}$ for non-base arcs is as follows.

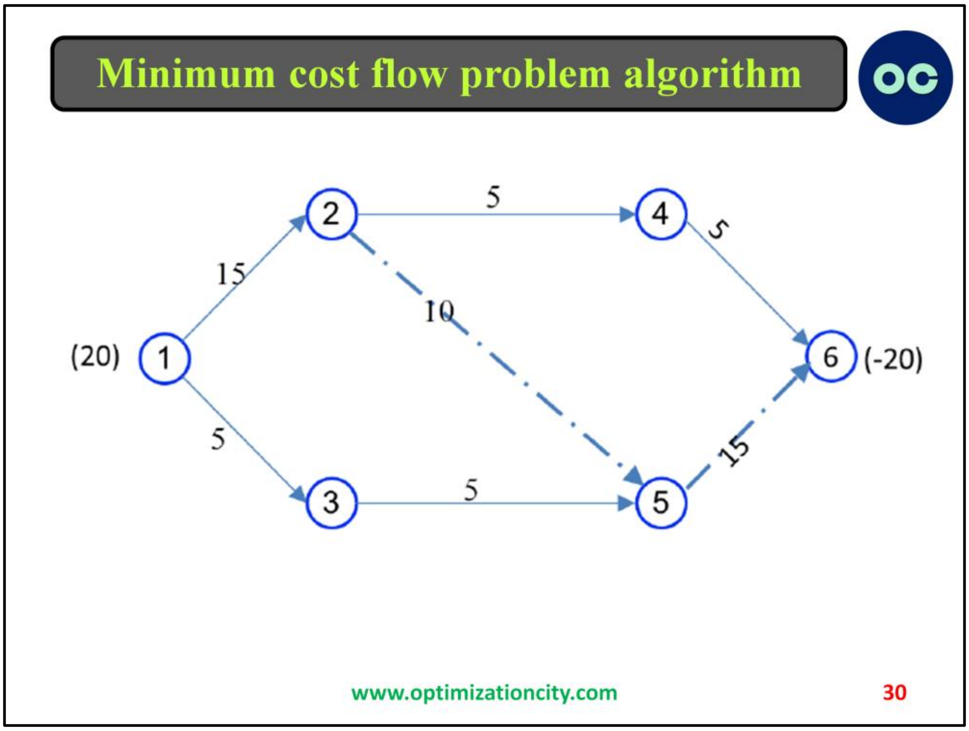
$$\overline{c_{13}} = 4 - y_1 + y_3 = 4 - 1 + 7 = 10$$
$$\overline{c_{25}} = 2 - y_2 + y_5 = 2 - 0 + 1 = 3$$
$$\overline{c_{45}} = 8 - y_4 + y_5 = 8 + 5 + 1 = 14$$
$$\overline{c_{56}} = 2 - y_5 + y_6 = 2 - 1 - 8 = -7$$

$$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$$

The arc (1,3) can enter the basic solutions.



By setting θ to 10, the arc (3,2) is removed from the base, and the arc (1,3) is added to the base. Therefore, the current solution is as follows.



In this solution, the objective function has a value of 155. Assuming y2 is zero, we can calculate the yi values for the basic arcs using the equations cij - yi + yj = 0 as follows.

Minimum cost flow problem algorithm

$$\overline{c_{12}} = 1 - y_1 + y_2 = 0 \rightarrow y_1 = 1$$
$$\overline{c_{13}} = 4 - y_1 + y_3 = 0 \rightarrow y_3 = -3$$
$$\overline{c_{35}} = 6 - y_3 + y_5 = 0 \rightarrow y_5 = -5$$
$$\overline{c_{24}} = 5 - y_2 + y_4 = 0 \rightarrow y_4 = -5$$
$$\overline{c_{46}} = 3 - y_4 + y_6 = 0 \rightarrow y_6 = -8$$

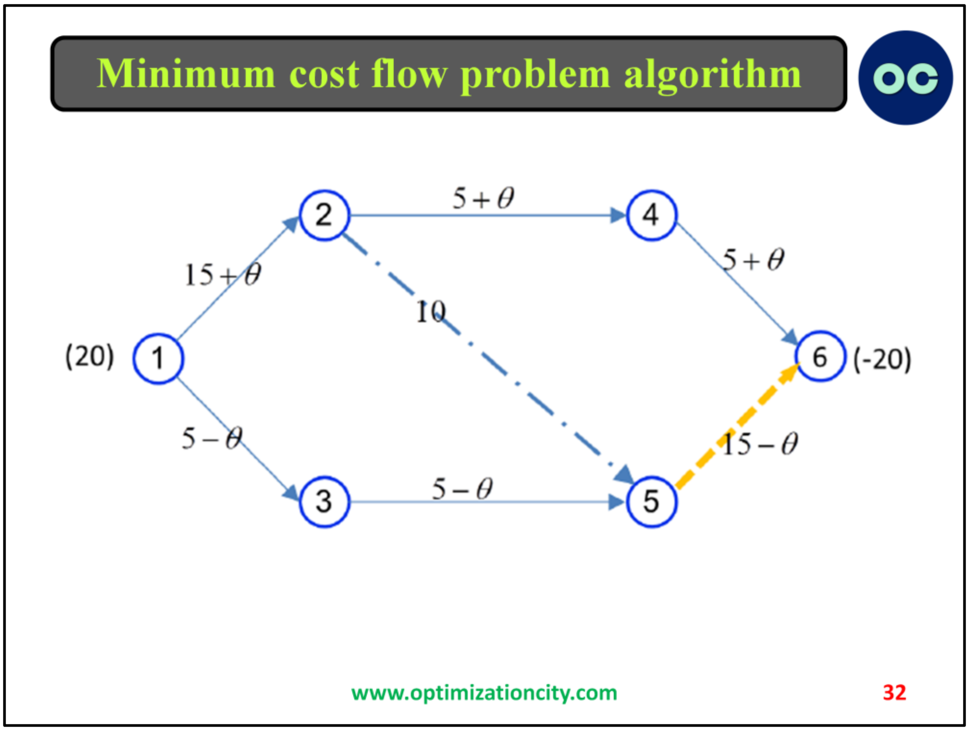
The $\overline{c_{ij}}$ for non-base arcs is as follows.

$$\overline{c_{32}} = 7 - y_1 + y_3 = 7 + 3 + 0 = 10$$
$$\overline{c_{25}} = 2 - y_2 + y_5 = 2 - 0 - 9 = -7$$
$$\overline{c_{45}} = 8 - y_4 + y_5 = 8 + 5 - 9 = 4$$
$$\overline{c_{56}} = 2 - y_5 + y_6 = 2 + 9 - 8 = 3$$

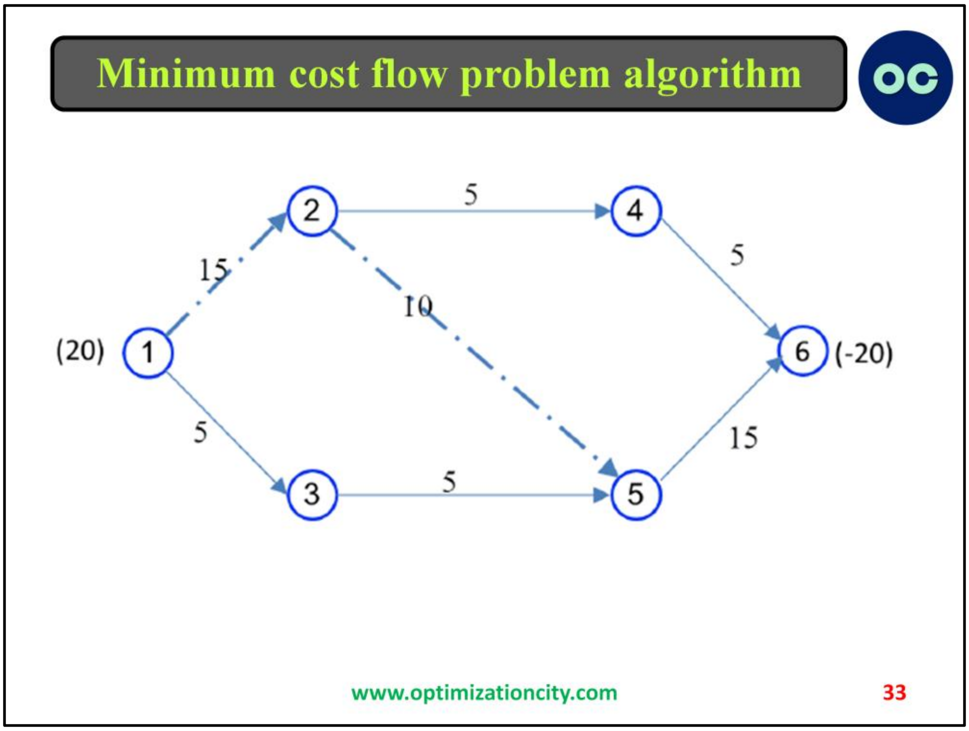
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$$

The arc (5,6) enters the base. The output arc is calculated as follows.

The arc (5,6) is added to the base. The calculation for the output arc is as follows.



By setting θ to 0, the arc (1,2) is removed from the base. Therefore, the updated current solution is as follows.



In this solution, the objective function has a value of 155. Assuming y2 is zero, we can calculate the yi values for the basic arcs using the equations cij - yi + yj = 0 as follows.

Minimum cost flow problem algorithm

$$c_{24} = 5 - y_2 + y_4 = 0 \rightarrow y_4 = -5$$
$$\overline{c_{46}} = 3 - y_4 + y_6 = 0 \rightarrow y_6 = -8$$
$$\overline{c_{56}} = 2 - y_5 + y_6 = 0 \rightarrow y_5 = -6$$
$$\overline{c_{35}} = 6 - y_3 + y_5 = 0 \rightarrow y_3 = 0$$
$$\overline{c_{13}} = 4 - y_1 + y_3 = 0 \rightarrow y_1 = 4$$

The $\overline{c_{ij}}$ for non-base arcs is as follows.

$$\overline{c_{12}} = 1 - y_1 + y_2 = -3$$
$$\overline{c_{32}} = 7 - y_3 + y_2 = 7$$
$$\overline{c_{25}} = 2 - y_2 + y_5 = -4$$
$$\overline{c_{45}} = 8 - y_4 + y_5 = 8 + 5 - 6 = 7$$

$$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$$

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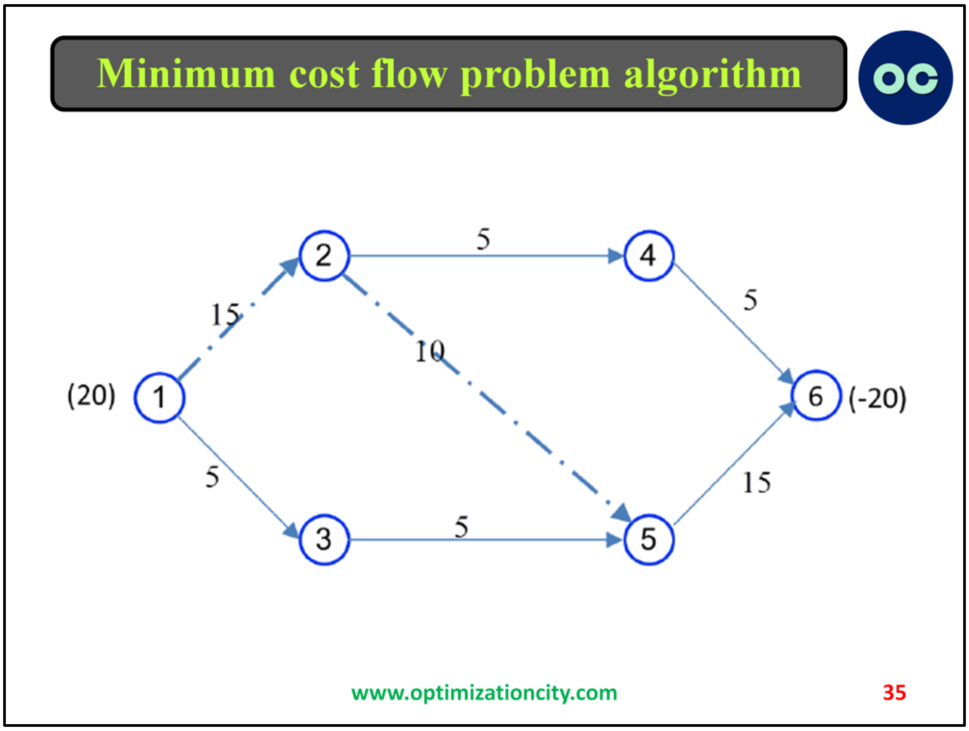
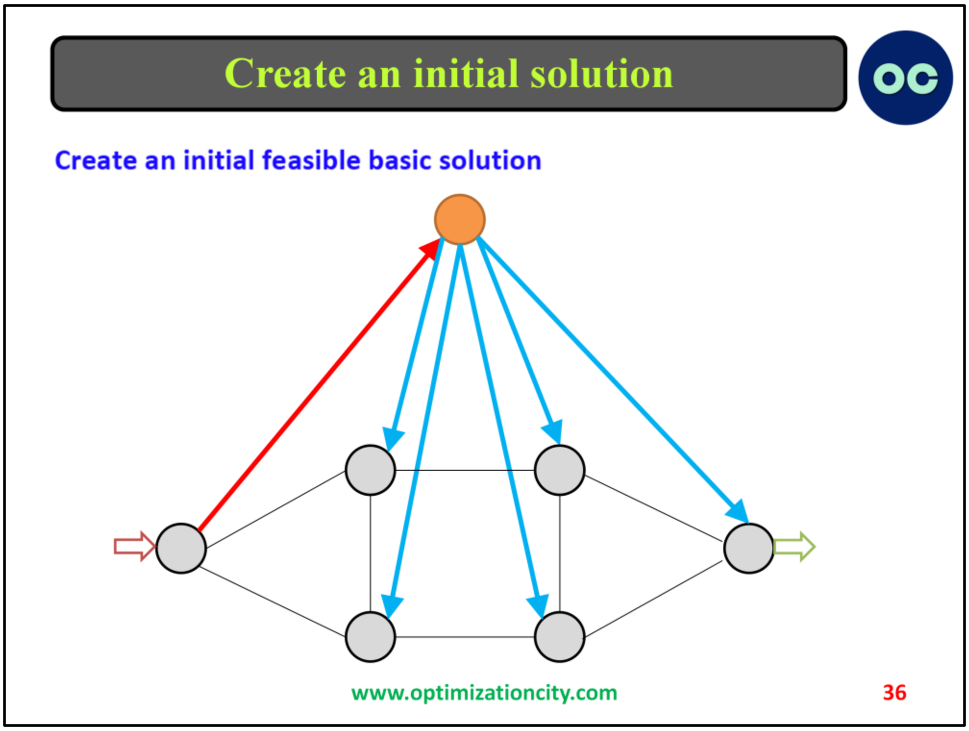
The cost cijbar for the non-basic arcs are as follows.

The optimality condition is met, indicating that we have reached the optimal solution.



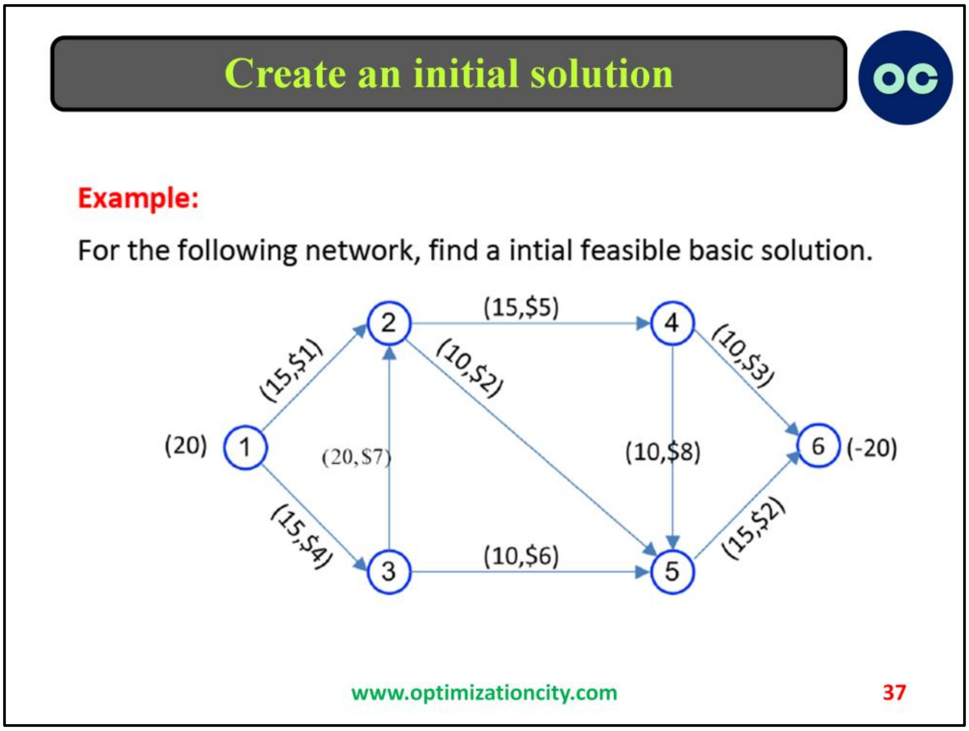
Note: Since θ is set to 0 in the current step, it implies a degenerate state where no improvement in the objective function value has occurred. The optimal solution is as follows.



To create an initial feasible basic solution, we can use a method when it's challenging to find such a solution, especially for large networks. This method involves introducing an additional node called Vn+1.

For nodes with a supply, we connect them to Vn+1 by adding an arc with a cost of one. On the other hand, for nodes with demand or intermediate nodes (nodes with neither supply nor demand), we connect Vn+1 to those nodes with an arc having a cost of one. The remaining arcs have a cost of zero.

Next, we apply the simplex algorithm to this network until the value of the optimal objective function becomes zero. Once we reach this point, we have obtained an initial feasible basic solution, and we can proceed with the simplex algorithm using this solution as a starting point.

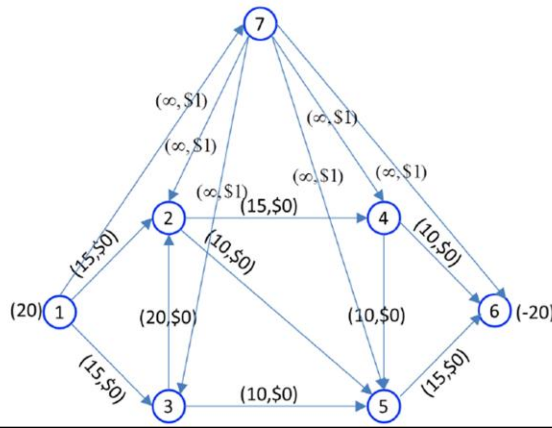


To find an initial feasible basic solution for the given network, we can follow these steps:

Create an initial solution

Solution:

We create an artificial node 7 and consider an arc from node 1 to 7 and from 7 to other nodes with a cost of 1.



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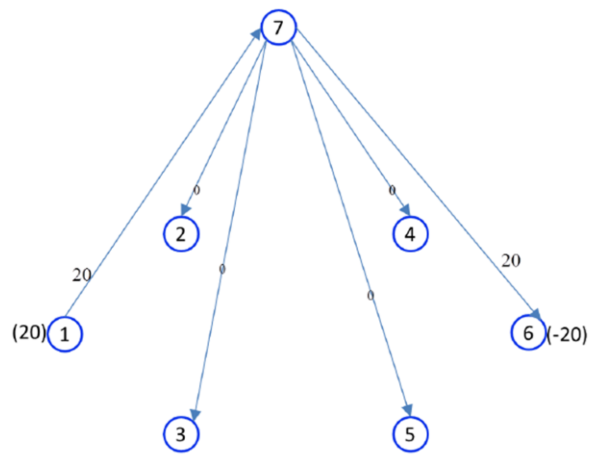
Create an artificial node, let's call it node 7.
Connect node 1 to node 7 with an arc, and create arcs from node 7 to all other nodes with a cost of 1.

Create an initial solution

The basic solution is as follows:



The resulting basic solution is as follows:
There is an arc from node 1 to node 7.
There are arcs from node 7 to each of the other nodes, namely nodes 2, 3, 4, 5, and 6, all with a cost of 1.

Create an initial solution

Assuming $y_7=0$, the y_i coefficients can be obtained based on $c_{ij}-y_i+y_j=0$.

$$\overline{c_{17}}=1-y_1+y_7=0\rightarrow y_1=1$$
$$\overline{c_{72}}=1-y_7+y_2=0\rightarrow y_2=-1$$
$$\overline{c_{73}}=1-y_7+y_3=0\rightarrow y_3=-1$$
$$\overline{c_{74}}=1-y_7+y_4=0\rightarrow y_4=-1$$
$$\overline{c_{75}}=1-y_7+y_5=0\rightarrow y_5=-1$$
$$\overline{c_{76}}=1-y_7+y_6=0\rightarrow y_6=-1$$

Assuming the value of y7 is zero, we can calculate the yi coefficients based on the equation cij - yi + yj = 0.

Create an initial solution

The cost $\overline{c_{ij}}$ for non-basic arcs is as follows.

$\overline{c_{12}} = 0 - y_1 + y_2 = -2 \Leftarrow$

$\overline{c_{13}} = 0 - y_1 + y_3 = -2$

$\overline{c_{32}} = 0 - y_3 + y_2 = 0$

$\overline{c_{24}} = 0 - y_2 + y_4 = 0$

$\overline{c_{25}} = 0 - y_2 + y_5 = 0$

$\overline{c_{35}} = 0 - y_3 + y_5 = 0$

$\overline{c_{45}} = 0 - y_4 + y_5 = 0$

$\overline{c_{46}} = 0 - y_4 + y_6 = 0$

$\overline{c_{56}} = 0 - y_5 + y_6 = 0$

$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$

$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$

$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$

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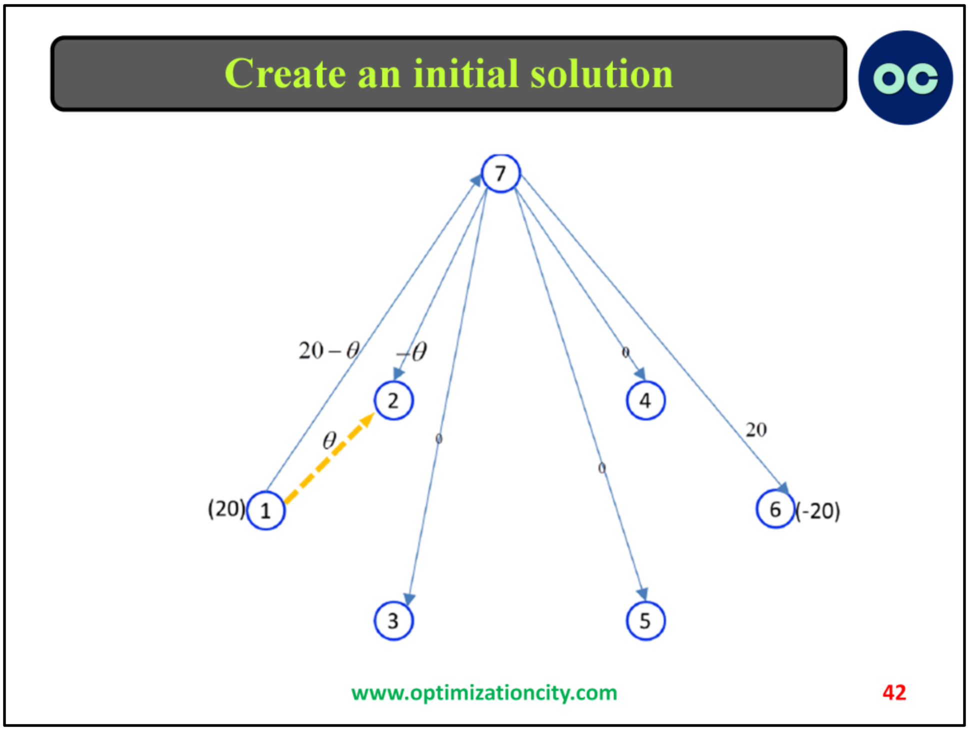
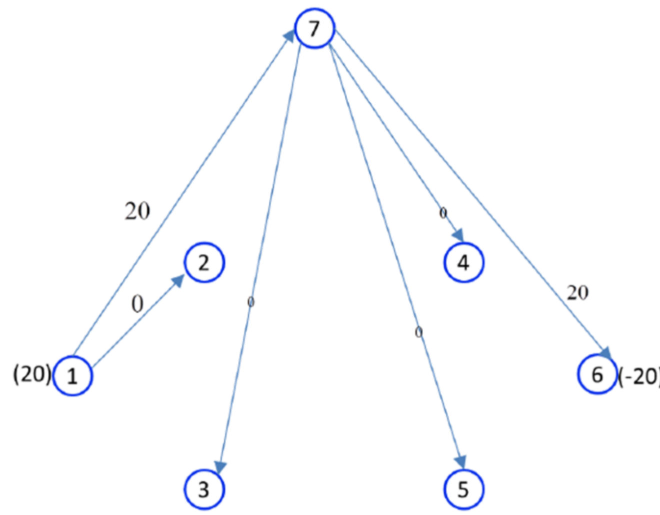
The cost cij values for non-basic arcs are as follows.

Since the arc (1,2) is at its lower bound and satisfies the condition, it enters the base. The output arc from the base can be determined as follows.



The arc (7,2) is removed from the base, resulting in the current basic solution as follows.

Create an initial solution



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In this iteration, the value of the objective function is 40. Assuming y7 is zero, we can calculate the values of the yi coefficients using the equation cij - yi + yj = 0.

Create an initial solution

$c_{17} = 1 - y_1 + y_7 = 0 \rightarrow y_1 = 1$

$\overline{c_{12}} = 0 - y_1 + y_2 = 0 \rightarrow y_2 = 1$

$\overline{c_{73}} = 1 - y_7 + y_3 = 0 \rightarrow y_3 = -1$

$\overline{c_{74}} = 1 - y_7 + y_4 = 0 \rightarrow y_4 = -1$

$\overline{c_{75}} = 1 - y_7 + y_5 = 0 \rightarrow y_5 = -1$

$\overline{c_{76}} = 1 - y_7 + y_6 = 0 \rightarrow y_6 = -1$

The cost $\overline{c_{ij}}$ for non-basic arcs is as follows.

$\overline{c_{72}} = 1 - y_7 + y_2 = 2$

$\overline{c_{13}} = 1 - y_1 + y_3 = -2 \Leftarrow$

$\overline{c_{32}} = 0 - y_3 + y_2 = 2$

$\overline{c_{24}} = 0 - y_2 + y_4 = -2$

$\overline{c_{25}} = 0 - y_2 + y_5 = -2$

$\overline{c_{35}} = 0 - y_3 + y_5 = 0$

$\overline{c_{45}} = 0 - y_4 + y_5 = 0$

$\overline{c_{46}} = 0 - y_4 + y_6 = 0$

$\overline{c_{56}} = 0 - y_5 + y_6 = 0$

$$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$$

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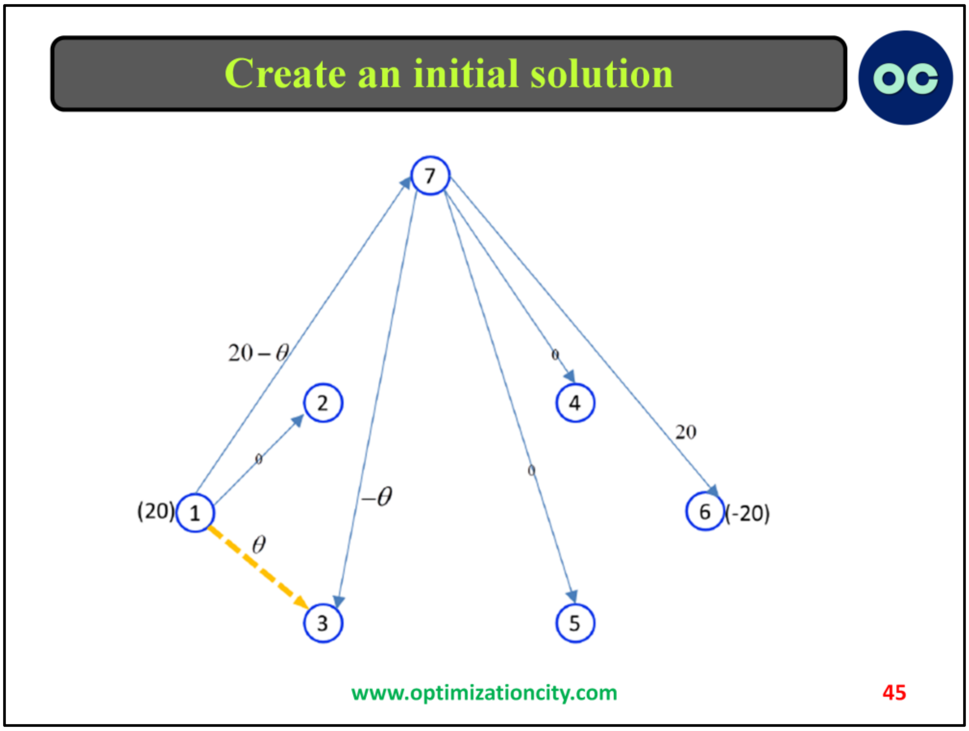
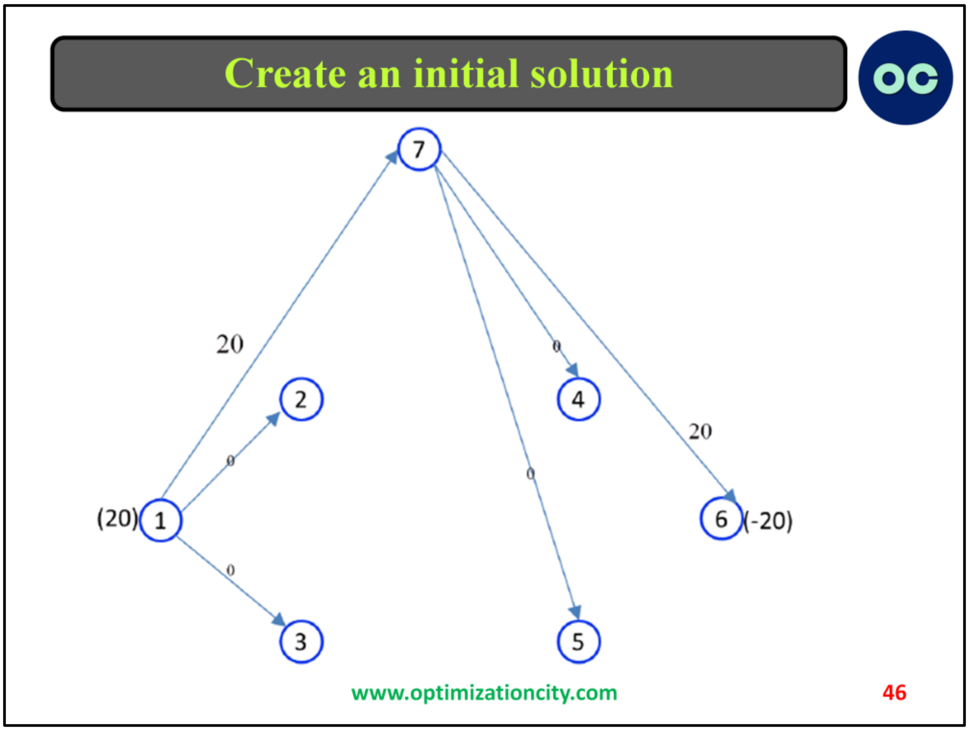
The cost cij values for non-basic arcs are as follows.

Since the arc (1,3) is at its lower bound and satisfies the condition, it enters the base. The output arc from the base can be determined as follows.



The arc (7,3) is removed from the base, resulting in the current basic solution as follows.



In this iteration, the value of the objective function is 40. Assuming y7 is zero, we can calculate the values of the yi coefficients using the equation cij - yi + yj = 0.

Create an initial solution

$\overline{c_{17}} = 1 - y_1 + y_7 = 0 \rightarrow y_1 = 1$

$\overline{c_{12}} = 0 - y_1 + y_2 = 0 \rightarrow y_2 = 1$

$\overline{c_{13}} = 0 - y_1 + y_3 = 0 \rightarrow y_3 = 1$

$\overline{c_{74}} = 1 - y_7 + y_4 = 0 \rightarrow y_4 = -1$

$\overline{c_{75}} = 1 - y_7 + y_5 = 0 \rightarrow y_5 = -1$

$\overline{c_{76}} = 1 - y_7 + y_6 = 0 \rightarrow y_6 = -1$

The cost $\overline{c_{ij}}$ for non-basic arcs is as follows.

$\overline{c_{72}} = 1 - y_7 + y_2 = 2$

$\overline{c_{73}} = 1 - y_7 + y_3 = 2$

$\overline{c_{32}} = 0 - y_3 + y_2 = 0$

$\overline{c_{24}} = 0 - y_2 + y_4 = -2 \Leftarrow$

$\overline{c_{25}} = 0 - y_2 + y_5 = -2$

$\overline{c_{35}} = 0 - y_3 + y_5 = -2$

$\overline{c_{45}} = 0 - y_4 + y_5 = 0$

$\overline{c_{46}} = 0 - y_4 + y_6 = 0$

$\overline{c_{56}} = 0 - y_5 + y_6 = 0$

$$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$$

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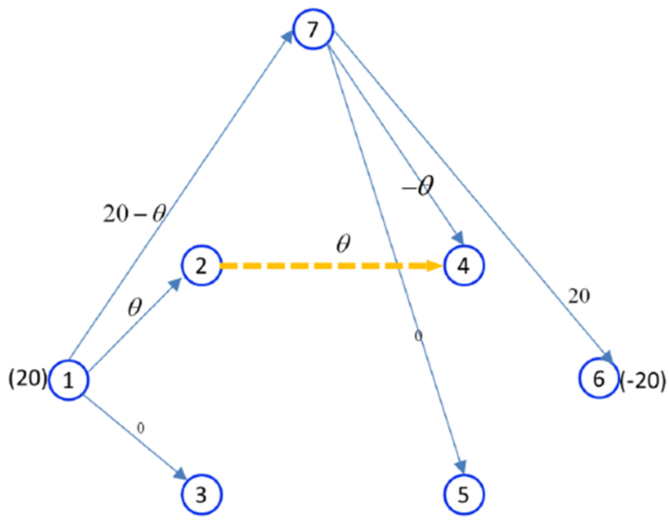
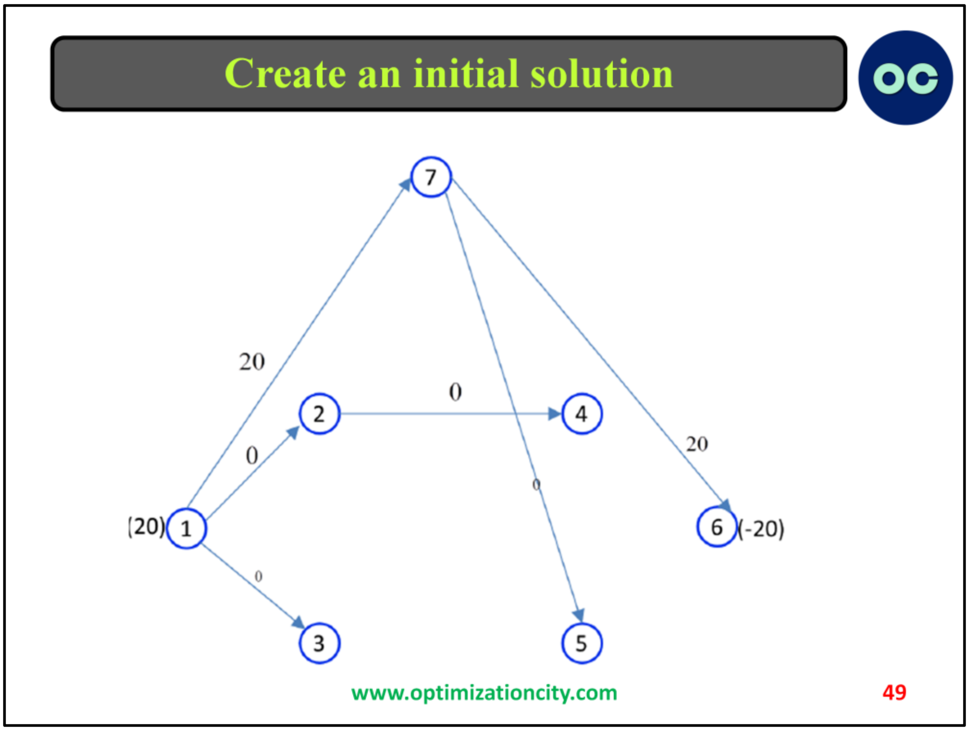
The cost cij values for non-basic arcs are as follows.

Since the arc (2,4) is at its lower bound and satisfies the condition, it enters the base. The output arc from the base can be determined as follows.

Create an initial solution





The arc (7,4) is removed from the base, resulting in the current basic solution as follows.

Create an initial solution

In this iteration, the value of the objective function is equal to 40. Assuming y_7=0, the y_is can be obtained based on c_{ij}-y_i+y_j=0.

$$\overline{c_{12}} = 0 - y_1 + y_2 = 0 \rightarrow y_2 = 1$$
$$\overline{c_{13}} = 0 - y_1 + y_3 = 0 \rightarrow y_3 = 1$$
$$\overline{c_{24}} = 0 - y_2 + y_4 = 0 \rightarrow y_4 = 1$$
$$\overline{c_{17}} = 1 - y_1 + y_7 = 0 \rightarrow y_1 = 1$$
$$\overline{c_{75}} = 1 - y_7 + y_5 = 0 \rightarrow y_5 = -1$$
$$\overline{c_{76}} = 1 - y_7 + y_6 = 0 \rightarrow y_6 = -1$$

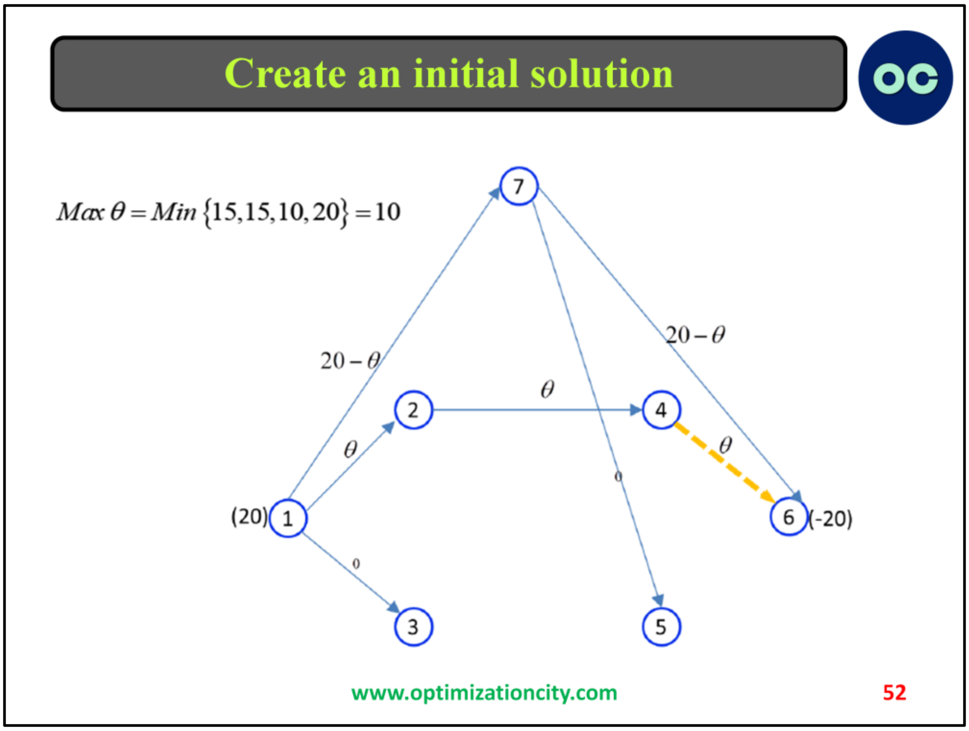
In this iteration, the objective function has a value of 40. Assuming y7 is zero, we can find the values of the yi coefficients by using the equation cij - yi + yj = 0.

Create an initial solution

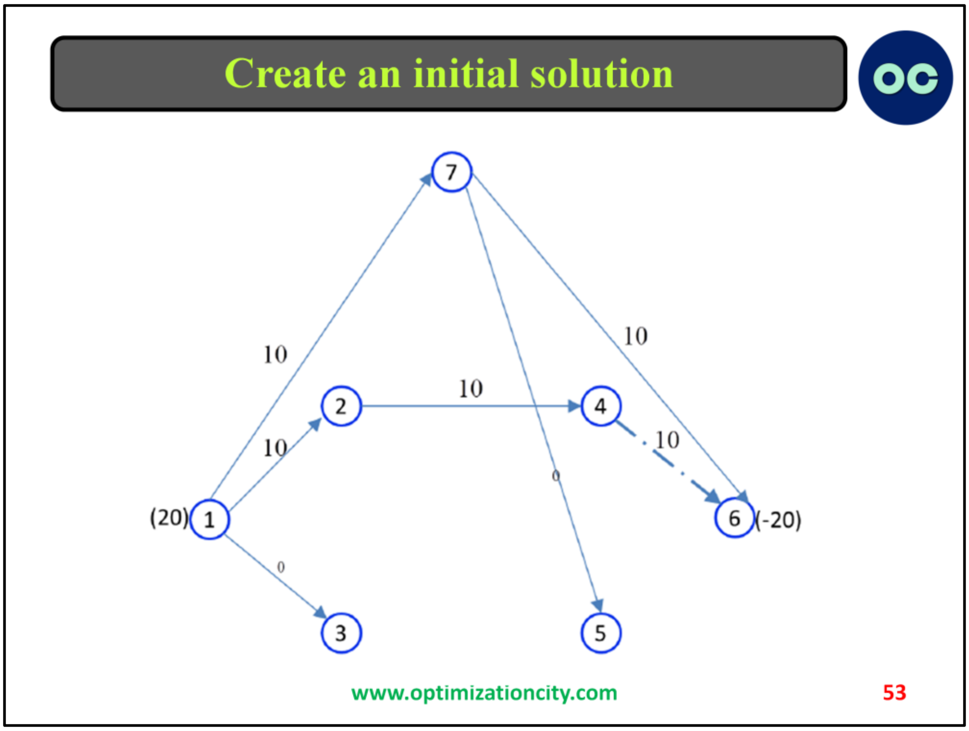
The cost $\overline{c_{ij}}$ for non-basic arcs is as follows.

$$\overline{c_{74}} = 1 - y_7 + y_4 = 2$$
$$\overline{c_{75}} = 1 - y_7 + y_5 = 0$$
$$\overline{c_{76}} = 1 - y_7 + y_6 = 0$$
$$\overline{c_{32}} = 0 - y_3 + y_2 = 0$$
$$\overline{c_{25}} = 0 - y_2 + y_5 = -2$$
$$\overline{c_{35}} = 0 - y_3 + y_5 = -2$$
$$\overline{c_{43}} = 0 - y_4 + y_3 = 0$$
$$\overline{c_{46}} = 0 - y_4 + y_6 = -2 \Leftarrow$$
$$\overline{c_{56}} = 0 - y_5 + y_6 = 0$$

$$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$$



Since the arc (4,6) is at its lower bound and meets the condition, it becomes part of the base. Let's determine the output arc from the base as follows.



With θ (theta) equal to 10, the arc (4,6) enters the base but quickly reaches its upper limit and is removed from the base. As a result of this change, the current solution is updated as follows, and the objective value becomes 20.

Create an initial solution

$\overline{c_{12}} = 0 - y_1 + y_2 = 0 \rightarrow y_2 = 1$

$\overline{c_{13}} = 0 - y_1 + y_3 = 0 \rightarrow y_3 = 1$

$\overline{c_{24}} = 0 - y_2 + y_4 = 0 \rightarrow y_4 = 1$

$\overline{c_{17}} = 1 - y_1 + y_7 = 0 \rightarrow y_1 = 1$

$\overline{c_{75}} = 1 - y_7 + y_5 = 0 \rightarrow y_5 = -1$

$\overline{c_{76}} = 1 - y_7 + y_6 = 0 \rightarrow y_6 = -1$

The cost $\overline{c_{ij}}$ for non-basic arcs is as follows.

$\overline{c_{74}} = 1 - y_7 + y_4 = 2$

$\overline{c_{75}} = 1 - y_7 + y_5 = 0$

$\overline{c_{76}} = 1 - y_7 + y_6 = 0$

$\overline{c_{32}} = 0 - y_3 + y_2 = 0$

$\overline{c_{25}} = 0 - y_2 + y_5 = -2$

$\overline{c_{35}} = 0 - y_3 + y_5 = -2 \Leftarrow$

$\overline{c_{43}} = 0 - y_4 + y_3 = 0$

$\overline{c_{46}} = 0 - y_4 + y_6 = -2$

$\overline{c_{56}} = 0 - y_5 + y_6 = 0$

$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$

$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$

$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$

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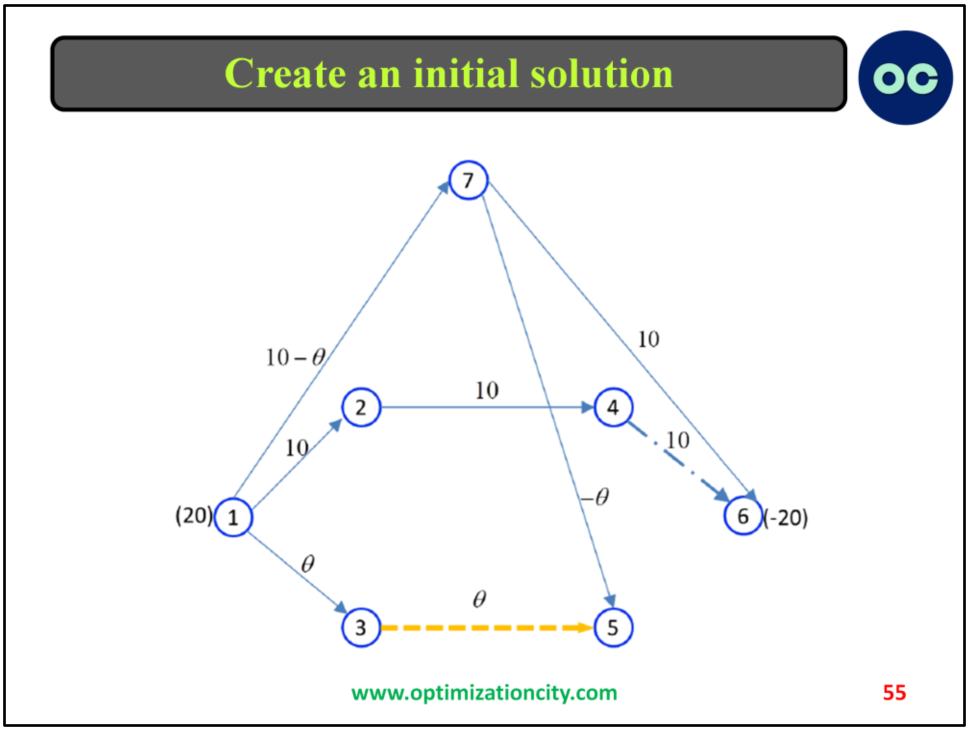
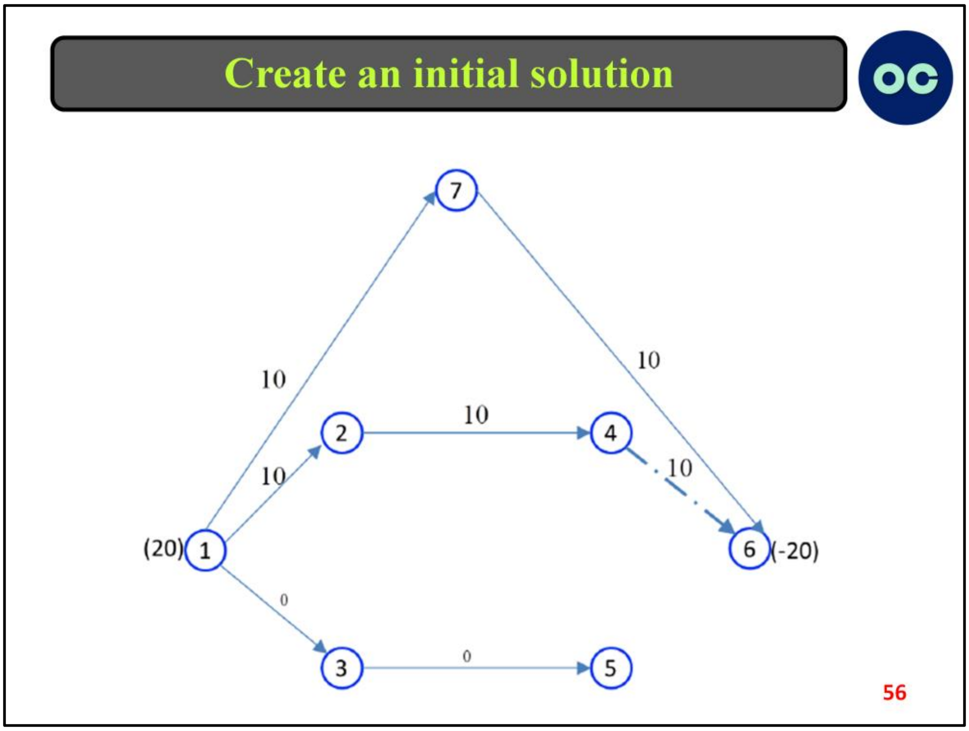
If we assume that y7 is zero, we can calculate the values of the yi coefficients using the equation cij - yi + yj = 0.

Now, let's consider the costs for non-basic arcs, which are as follows.



The arc (4,6) has reached its maximum limit and fulfills the condition, making it an optimal arc. Conversely, the arc (3,5) is at its minimum limit and meets the condition, allowing it to enter the base. Let's find the output arc from the base using the following steps.



The arc (7,5) is no longer part of the base, which leads to the current basic solution as follows. Moreover, the objective value remains at 20.

Create an initial solution

Assuming y_7=0, the y_is can be obtained based on c_{ij}-y_i+y_j=0.

$\overline{c_{17}} = 1 - y_1 + y_7 = 0 \rightarrow y_1 = 1$

$\overline{c_{12}} = 0 - y_1 + y_2 = 0 \rightarrow y_2 = 1$

$\overline{c_{13}} = 0 - y_1 + y_3 = 0 \rightarrow y_3 = 1$

$\overline{c_{35}} = 0 - y_3 + y_5 = 0 \rightarrow y_5 = 1$

$\overline{c_{24}} = 0 - y_2 + y_4 = 0 \rightarrow y_4 = 1$

$\overline{c_{76}} = 1 - y_7 + y_6 = 0 \rightarrow y_6 = -1$

The cost $\overline{c_{ij}}$ for non-basic arcs is as follows.

$\overline{c_{72}} = 1 - y_7 + y_2 = 2$

$\overline{c_{73}} = 1 - y_7 + y_3 = 2$

$\overline{c_{74}} = 1 - y_7 + y_4 = 2$

$\overline{c_{75}} = 1 - y_7 + y_5 = 2$

$\overline{c_{32}} = 0 - y_3 + y_2 = 0$

$\overline{c_{25}} = 0 - y_2 + y_5 = 0$

$\overline{c_{35}} = 0 - y_3 + y_5 = 0$

$\overline{c_{45}} = 0 - y_4 + y_5 = 0$

$\overline{c_{46}} = 0 - y_4 + y_6 = -2$

$\overline{c_{56}} = 0 - y_5 + y_6 = -2 \Leftarrow$

$$\overline{c_{ij}} = c_{ij} - y_i + y_j \geq 0; x_{ij} = l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j = 0; l_{ij} < x_{ij} < l_{ij}$$
$$\overline{c_{ij}} = c_{ij} - y_i + y_j \leq 0; x_{ij} = u_{ij}$$

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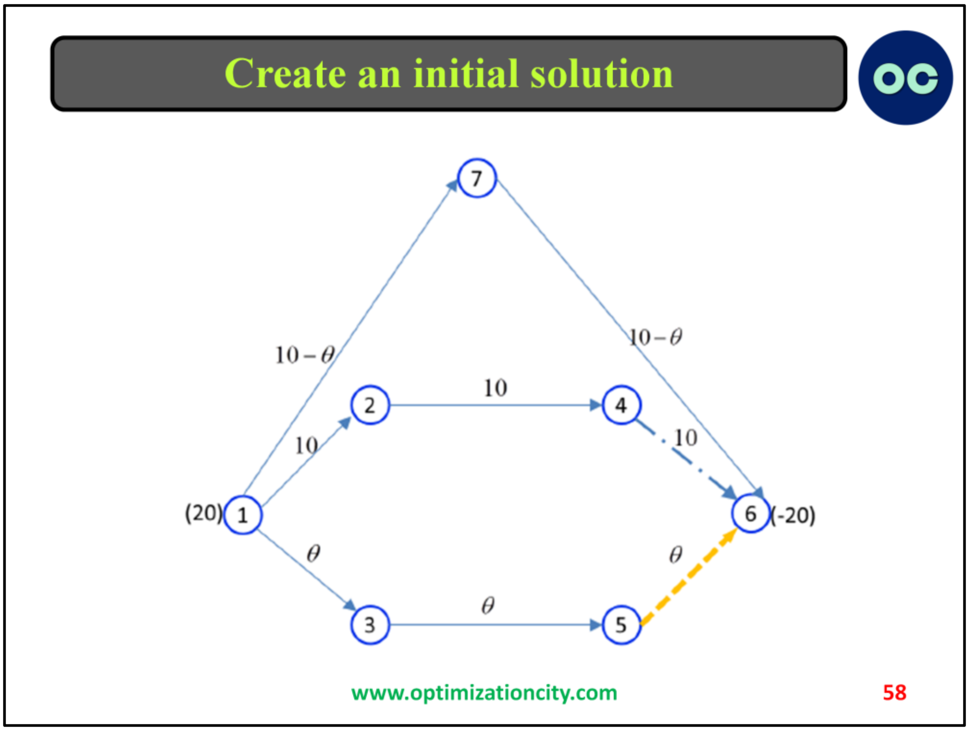
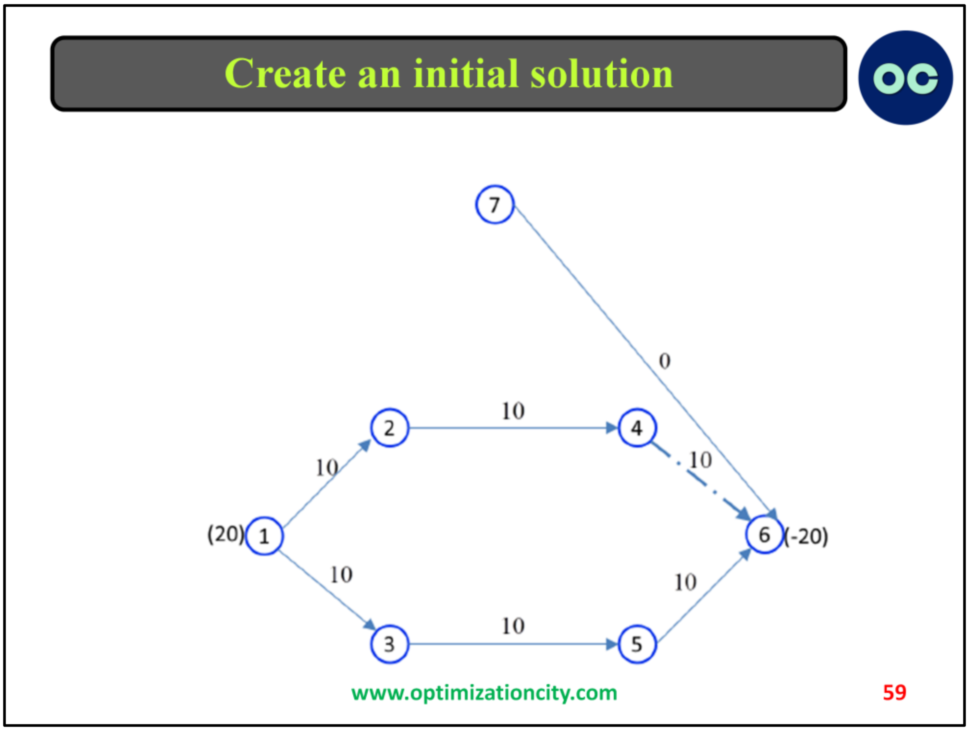
If we assume that y7 is zero, we can calculate the values of the yi coefficients using the equation cij - yi + yj = 0.

Now, let's consider the costs for non-basic arcs, which are as follows.



Since the arc (5,6) is at its lower bound and meets the condition, it enters the base. Let's determine the output arc from the base as follows.



The arc (1,7) is removed from the base, resulting in the current basic solution as follows. Additionally, the objective value is equal to 0.

Create an initial solution

Assuming $y_7=0$, the y_is can be obtained based on $c_{ij}-y_i+y_j=0$.

$$\overline{c_{12}}=0-y_1+y_2=0 \rightarrow y_2=-1$$
$$\overline{c_{13}}=0-y_1+y_3=0 \rightarrow y_1=-1$$
$$\overline{c_{35}}=0-y_3+y_5=0 \rightarrow y_3=-1$$
$$\overline{c_{24}}=0-y_2+y_4=0 \rightarrow y_4=-1$$
$$\overline{c_{56}}=0-y_5+y_6=0 \rightarrow y_5=-1$$
$$\overline{c_{76}}=1-y_7+y_6=0 \rightarrow y_6=-1$$

The cost $\overline{c_{ij}}$ for non-basic arcs is as follows.

$$\overline{c_{17}}=1-y_1+y_7=2$$
$$\overline{c_{72}}=1-y_7+y_2=2$$
$$\overline{c_{73}}=1-y_7+y_3=2$$
$$\overline{c_{74}}=1-y_7+y_4=2$$
$$\overline{c_{75}}=1-y_7+y_5=2$$
$$\overline{c_{32}}=0-y_3+y_2=0$$
$$\overline{c_{25}}=0-y_2+y_5=0$$
$$\overline{c_{45}}=0-y_4+y_5=0$$
$$\overline{c_{46}}=0-y_4+y_6=-2$$

$$\overline{c_{ij}}=c_{ij}-y_i+y_j\geq 0; x_{ij}=l_{ij}$$
$$\overline{c_{ij}}=c_{ij}-y_i+y_j=0; l_{ij}<x_{ij}<l_{ij}$$
$$\overline{c_{ij}}=c_{ij}-y_i+y_j\leq 0; x_{ij}=u_{ij}$$

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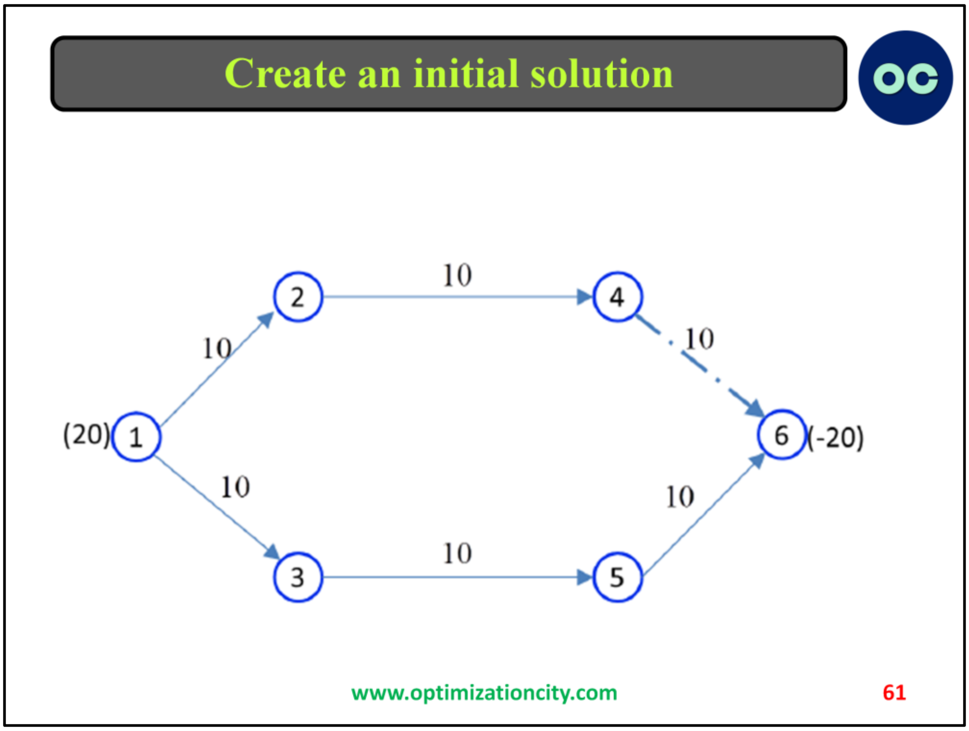
Assuming y7 is zero, we can calculate the values of the yi coefficients using the equation cij - yi + yj = 0.
Now, let's consider the costs for non-basic arcs, which are as follows.
The optimal conditions are met, indicating that the initial feasible basic solution is as follows. It is important to note that the optimal objective function value is zero.



Therefore, there is no need for artificial arcs anymore, and node 7 can be removed from the network. Additionally, the arc (7,6) is deleted as well.

Exercises

More exercises

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If you want more practice to improve what you've learned, I've got something perfect for you. Check out my website where you'll find plenty of extra exercises to explore. Take a moment to visit my website and try out the exercises. They're not only educational but also enjoyable. I'm confident they'll help you strengthen your understanding and feel more confident. Give it a try and see for yourself.

Thanks

How to find us

www.optimizationcity.com

optimizationcity@gmail.com